7 WORK, ENERGY, AND ENERGY RESOURCES

Figure 7.1 How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Chapter Outline

Connection for AP® Courses

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods to the energy we use to run our cars and the sunlight that warms us on the beach. You can also cite examples of what people call “energy” that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics.

There is no simple and accurate scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define energy as the ability to do **work**, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form. The work-energy theorem supports Big Idea 3, that interactions between objects are described by forces. In particular, exerting a force on an object may do work on it, changing it's energy (Enduring Understanding 3.E). The work-energy theorem, introduced in this chapter, establishes the relationship between work done on an object by an external force and changes in the object’s kinetic energy (Essential Knowledge 3.E.1).

Similarly, systems can do work on each other, supporting Big Idea 4, that interactions between systems can result in changes in those systems—in this case, changes in the total energy of the system (Enduring Understanding 4.C). The total energy of the system is the sum of its kinetic energy, potential energy, and microscopic internal energy (Essential Knowledge 4.C.1). In this chapter students learn how to calculate kinetic, gravitational, and elastic potential energy in order to determine the total mechanical energy of a system. The transfer of mechanical energy into or out of a system is equal to the work done on the system by an external force with a nonzero component parallel to the displacement (Essential Knowledge 4.C.2).

An important aspect of energy is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is "conserved." **Conservation of energy** (as physicists call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$). This is one of the most important applications of Big Idea 5, that changes that occur as a result of interactions are constrained by conservation laws. Specifically, there are many situations where conservation of energy (Enduring Understanding 5.B) is both a useful concept and starting point for calculations related to the system. Note, however, that conservation doesn't necessarily mean that energy in a system doesn't change. Energy may be transferred into or out of the system, and the change must be equal to the amount transferred (Enduring Understanding 5.A). This may occur if there is an external force or a transfer between external objects and the system (Essential Knowledge 5.A.3). Energy is one of the fundamental quantities that are conserved for all systems (Essential Knowledge 5.A.2). The chapter introduces concepts of kinetic energy and potential energy. Kinetic energy is introduced as an energy of motion that can be changed by the amount of work done by an external force. Potential energy can only exist when objects interact with each other via conservative forces according to classical physics (Essential Knowledge 5.B.3). Because of this, a single object can only have kinetic energy and no potential energy (Essential Knowledge 5.B.1). The chapter also introduces the idea that the energy transfer is equal to the work done on the system by external forces and the rate of energy transfer is defined as power (Essential Knowledge 5.B.5).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

The concepts in this chapter support:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.E A force exerted on an object can change the kinetic energy of the object.

Essential Knowledge 3.E.1 The change in the kinetic energy of an object depends on the force exerted on the object and on the displacement of the object during the interval that the force is exerted.

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding 4.C Interactions with other objects or systems can change the total energy of a system.

Essential Knowledge 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

Essential Knowledge 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Essential Knowledge 5.A.3 An interaction can be either a force exerted by objects outside the system or the transfer of some quantity with objects outside the system.

Enduring Understanding 5.B The energy of a system is conserved.

Essential Knowledge 5.B.1 Classically, an object can only have kinetic energy since potential energy requires an interaction between two or more objects.

Essential Knowledge 5.B.3 A system with internal structure can have potential energy. Potential energy exists within a system if the objects within that system interact with conservative forces.

Essential Knowledge 5.B.5 Energy can be transferred by an external force exerted on an object or system that moves the object or system through a distance; this energy transfer is called work. Energy transfer in mechanical or electrical systems may occur at different rates. Power is defined as the rate of energy transfer into, out of, or within a system.

7.1 Work: The Scientific Definition

Learning Objectives

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement of an object determine whether the work done on the object is positive, negative, or zero.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.B.5.1** The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance. **(S.P. 4.2, 5.1)**

- **5.B.5.2** The student is able to design an experiment and analyze graphical data in which interpretations of the area under a force-distance curve are needed to determine the work done on or by the object or system. **(S.P. 4.5, 5.1)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done on an object, a force must be exerted on that object and there must be displacement of that object in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion and the distance through which the force acts*. For a constant force, this is expressed in equation form as

$$W = \mid \mathbf{F} \mid (\cos\theta) \mid \mathbf{d} \mid , \tag{7.1}$$

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in Figure 7.2. We can also write this as

$$W = Fd\cos\theta. \tag{7.2}$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta, \tag{7.3}$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.

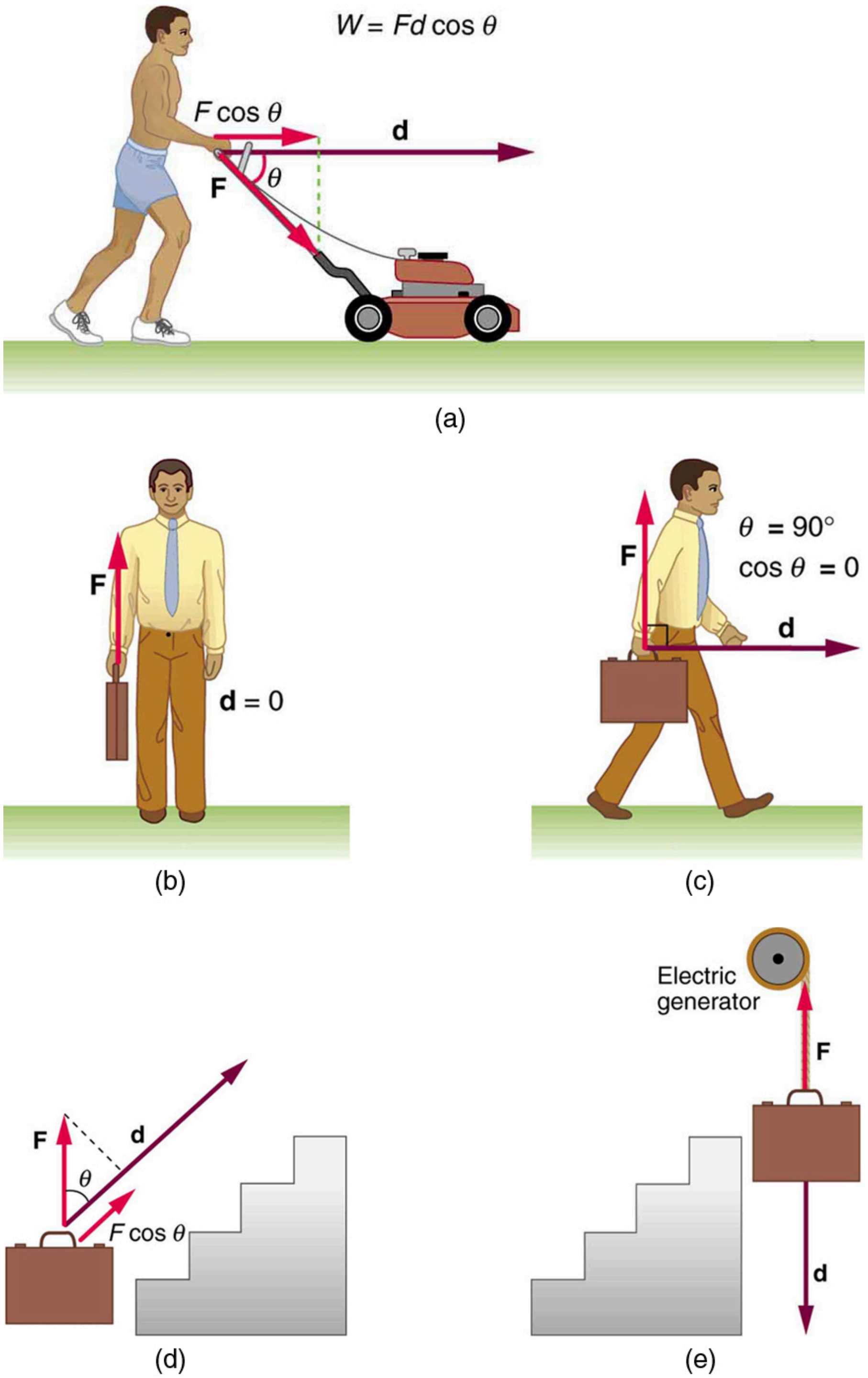

Figure 7.2 Examples of work. (a) The work done by the force $\mathbf{F}$ on this lawn mower is $Fd \cos \theta$. Note that $F \cos \theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force $\mathbf{F}$ in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because $\mathbf{F}$ and $\mathbf{d}$ are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in **Figure 7.2**. The person holding the briefcase in **Figure 7.2**(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see **Gravitational Potential Energy** for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in **Figure 7.2**(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in **Figure 7.2**(d), work *is* done—energy is transferred to the briefcase. Finally, in **Figure 7.2**(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^\circ$, and $\cos 180^\circ = -1$; therefore, W is negative.

Real World Connections: When Work Happens

Note that work as we define it is not the same as effort. You can push against a concrete wall all you want, but you won't move it. While the pushing represents effort on your part, the fact that you have not changed the wall's state in any way indicates that you haven't done work. If you did somehow push the wall over, this would indicate a change in the wall's state, and therefore you would have done work.

This can also be shown with **Figure 7.2**(a): as you push a lawnmower against friction, both you and friction are changing the lawnmower's state. However, only the component of the force parallel to the movement is changing the lawnmower's state. The component perpendicular to the motion is trying to push the lawnmower straight into Earth; the lawnmower does not move into Earth, and therefore the lawnmower's state is not changing in the direction of Earth.

Similarly, in **Figure 7.2**(c), both your hand and gravity are exerting force on the briefcase. However, they are both acting perpendicular to the direction of motion, hence they are not changing the condition of the briefcase and do no work. However, if the briefcase were dropped, then its displacement would be parallel to the force of gravity, which would do work on it, changing its state (it would fall to the ground).

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 7.1 Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in **Figure 7.2**(a) if he exerts a constant force of 75.0 N at an angle 35° below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of $10{,}000\text{ kJ}$ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C, and is equivalent to 4.184 J, while one *food calorie* (1 kcal) is equivalent to 4184 J.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd\cos\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd\cos\theta. \tag{7.4}$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\text{ N})(25.0\text{ m})\cos(35.0^\circ) \\ &= 1536\text{ J} = 1.54\times10^3\text{ J}. \end{aligned} \tag{7.5}$$

Converting the work in joules to kilocalories yields $W = (1536\text{ J})(1\text{ kcal}/4184\text{ J}) = 0.367\text{ kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\text{ kcal}} = 1.53\times10^{-4}. \tag{7.6}$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Applying the Science Practices: Boxes on Floors

Plan and design an experiment to determine how much work you do on a box when you are pushing it over different floor surfaces. Make sure your experiment can help you answer the following questions: What happens on different surfaces? What happens if you take different routes across the same surface? Do you get different results with two people pushing on perpendicular surfaces of the box? What if you vary the mass in the box? Remember to think about both your effort in any given instant (a proxy for force exerted) and the total work you do. Also, when planning your experiments, remember that in any given set of trials you should only change one variable.

You should find that you have to exert more effort on surfaces that will create more friction with the box, though you might be surprised by which surfaces the box slides across easily. Longer routes result in your doing more work, even though the box ends up in the same place. Two people pushing on perpendicular sides do less work for their total effort, due to the forces and displacement not being parallel. A more massive box will take more effort to move.

Applying the Science Practices: Force-Displacement Diagrams

Suppose you are given two carts and a track to run them on, a motion detector, a force sensor, and a computer that can record the data from the two sensors. Plan and design an experiment to measure the work done on one of the carts, and compare your results to the work-energy theorem. Note that the motion detector can measure both displacement and velocity versus time, while the force sensor measures force over time, and the carts have known masses. Recall that the work-energy theorem states that the work done on a system (force over displacement) should equal the change in kinetic energy. In your experimental design, describe and compare two possible ways to calculate the work done.

Sample Response: One possible technique is to set up the motion detector at one end of the track, and have the computer record both displacement and velocity over time. Then attach the force sensor to one of the carts, and use this cart, through the force sensor, to push the second cart toward the motion detector. Calculate the difference between the final and initial kinetic energies (the kinetic energies after and before the push), and compare this to the area of a graph of force versus displacement for the duration of the push. They should be the same.

7.2 Kinetic Energy and the Work-Energy Theorem

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.E.1.1** The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. **(S.P. 6.4, 7.2)**
- **3.E.1.2** The student is able to use net force and velocity vectors to determine qualitatively whether kinetic energy of an object would increase, decrease, or remain unchanged. **(S.P. 1.4)**
- **3.E.1.3** The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. **(S.P. 1.4, 2.2)**
- **3.E.1.4** The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. **(S.P. 2.2)**
- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.B.5.3** The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **(S.P. 1.5, 2.2, 6.4)**

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in **Figure 7.2**(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in **Figure 7.2**(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in **Figure 7.2**(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system.

Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton’s laws in **Dynamics: Force and Newton's Laws of Motion** that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F}_{\text{net}}$. In equation form, this is $W_{\text{net}} = F_{\text{net}} d \cos \theta$ where θ is the angle between the force vector and the displacement vector.

Figure 7.3(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos \theta$ vs. d graph. In this case, $F \cos \theta$ is constant. You can see that the area under the graph is $Fd \cos \theta$, or the work done. **Figure 7.3**(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos \theta)_{i(\text{ave})}$. The work done is $(F \cos \theta)_{i(\text{ave})} d_i$ for each strip, and the total work done is the sum of the W_i . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

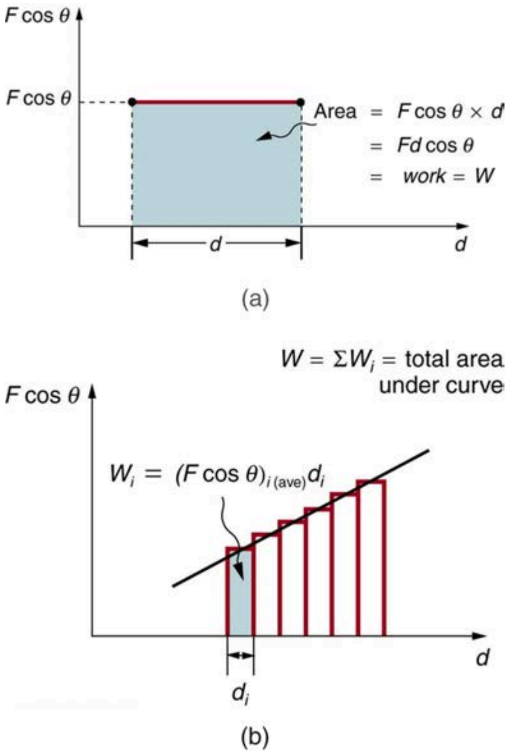

Figure 7.3 (a) A graph of $F \cos \theta$ vs. d , when $F \cos \theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F \cos \theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Real World Connections: Work and Direction

Consider driving in a car. While moving, you have forward velocity and therefore kinetic energy. When you hit the brakes, they exert a force opposite to your direction of motion (acting through the wheels). The brakes do work on your car and reduce the kinetic energy. Similarly, when you accelerate, the engine (acting through the wheels) exerts a force in the direction of motion. The engine does work on your car, and increases the kinetic energy. Finally, if you go around a corner at a constant speed, you have the same kinetic energy both before and after the corner. The force exerted by the engine was perpendicular to the direction of motion, and therefore did no work and did not change the kinetic energy.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in **Figure**

7.4.

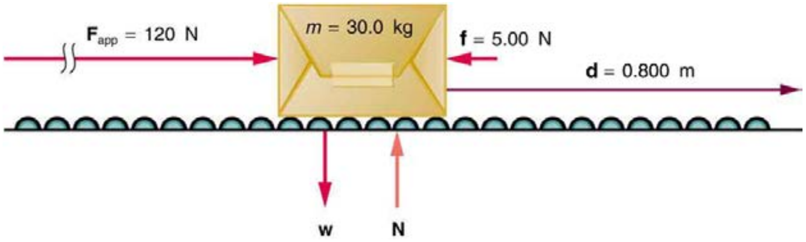

Figure 7.4 A package on a roller belt is pushed horizontally through a distance $\mathbf{d}$.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0^\circ$ and $\cos\theta = 1$, and the net work is given by

$$W_{\text{net}} = F_{\text{net}}d. \tag{7.7}$$

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v . The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See **Example 7.2**.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = ma$ from Newton's second law gives

$$W_{\text{net}} = mad. \tag{7.8}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in **Motion Equations for Constant Acceleration in One Dimension** for the change in speed over a distance d if the acceleration has the constant value a ; namely, $v^2 = v_0{}^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \dfrac{v^2 - v_0{}^2}{2d}$. When a is substituted into the preceding expression for W_{net} , we obtain

$$W_{\text{net}} = m\left(\frac{v^2 - v_0{}^2}{2d}\right)d. \tag{7.9}$$

The d cancels, and we rearrange this to obtain

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2\,. \tag{7.10}$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \tag{7.11}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v . (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2, \tag{7.12}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in **Figure 7.4**, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Applying the Science Practices: Cars on a Hill

Assemble a ramp suitable for rolling some toy cars up or down. Then plan a series of experiments to determine how the direction of a force relative to the velocity of an object alters the kinetic energy of the object. Note that gravity will be pointing down in all cases. What happens if you start the car at the top? How about at the bottom, with an initial velocity that is increasing? If your ramp is wide enough, what happens if you send the toy car straight across? Does varying the surface of the ramp change your results?

Sample Response: When the toy car is going down the ramp, with a component of gravity in the same direction, the kinetic energy increases. Sending the car up the ramp decreases the kinetic energy, as gravity is opposing the motion. Sending the car sideways should result in little to no change. If you have a surface that generates more friction than a smooth surface (carpet), note that the friction always opposed the motion, and hence decreases the kinetic energy.

Example 7.2 Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in **Figure 7.4** is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2. \tag{7.13}$$

Entering known values gives

$$\text{KE} = 0.5(30.0\text{ kg})(0.500\text{ m/s})^2, \tag{7.14}$$

which yields

$$\text{KE} = 3.75\text{ kg}\cdot\text{m}^2/\text{s}^2 = 3.75\text{ J}. \tag{7.15}$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Real World Connections: Center of Mass

Suppose we have two experimental carts, of equal mass, latched together on a track with a compressed spring between them. When the latch is released, the spring does 10 J of work on the carts (we'll see how in a couple of sections). The carts move relative to the spring, which is the center of mass of the system. However, the center of mass stays fixed. How can we consider the kinetic energy of this system?

By the work-energy theorem, the work done by the spring on the carts must turn into kinetic energy. So this system has 10 J of kinetic energy. The total kinetic energy of the system is the kinetic energy of the center of mass of the system relative to the fixed origin plus the kinetic energy of each cart relative to the center of mass. We know that the center of mass relative to the fixed origin does not move, and therefore all of the kinetic energy must be distributed among the carts relative to the center of mass. Since the carts have equal mass, they each receive an equal amount of kinetic energy, so each cart has 5.0 J of kinetic energy.

In our example, the forces between the spring and each cart are internal to the system. According to Newton's third law, these internal forces will cancel since they are equal and opposite in direction. However, this does not imply that these internal forces will not do work. Thus, the change in kinetic energy of the system is caused by work done by the force of the spring, and results in the motion of the two carts relative to the center of mass.

Example 7.3 Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in **Figure 7.4** with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See **Figure 7.4**.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\text{ N} - 5.00\text{ N} = 115\text{ N}$. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115\text{ N})(0.800\text{ m}) \\ &= 92.0\text{ N}\cdot\text{m} = 92.0\text{ J}. \end{aligned} \tag{7.16}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}}d\cos(0^\circ) = F_{\text{app}}d \\ &= (120\text{ N})(0.800\text{ m}) \\ &= 96.0\text{ J} \end{aligned} \tag{7.17}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^\circ$, and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}}d\cos(180^\circ) = -F_{\text{fr}}d \\ &= -(5.00\text{ N})(0.800\text{ m}) \\ &= -4.00\text{ J}. \end{aligned} \tag{7.18}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0\text{ J}, \\ W_{\text{fr}} &= -4.00\text{ J}. \end{aligned} \tag{7.19}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\text{ J}. \tag{7.20}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 7.4 Determining Speed from Work and Energy

Find the speed of the package in **Figure 7.4** at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0{}^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2. \tag{7.21}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0{}^2. \tag{7.22}$$

Thus,

$$\frac{1}{2}mv^2 = 92.0 \text{ J}+3.75 \text{ J} = 95.75 \text{ J}. \tag{7.23}$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75 \text{ J})}{m}} = \sqrt{\frac{191.5 \text{ kg} \cdot \text{m}^2/\text{s}^2}{30.0 \text{ kg}}} \\ &= 2.53 \text{ m/s}. \end{aligned} \tag{7.24}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 7.5 Work and Energy Can Reveal Distance, Too

How far does the package in **Figure 7.4** coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^\text{o}$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{\text{fr}} = -95.75 \text{ J}$. Furthermore, $W_{\text{fr}} = fd' \cos\theta = -fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75 \text{ J}}{5.00 \text{ N}}, \tag{7.25}$$

and so

$$d' = 19.2 \text{ m}. \tag{7.26}$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

7.3 Gravitational Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass *m* at height *h* on Earth is given by *PE*g = *mgh*.
- Show how knowledge of potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **5.B.1.1** The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. **(S.P. 1.4, 2.2)**
- **5.B.1.2** The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. **(S.P. 1.5)**

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in **Figure 7.5**. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of PE_g to KE without explicitly considering the intermediate step of work. (See **Example 7.7**.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

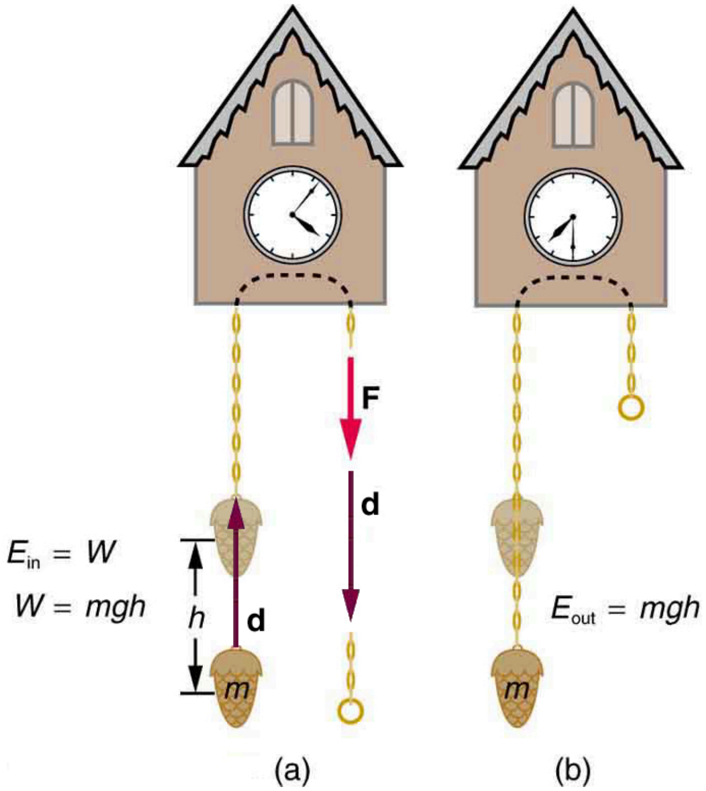

Figure 7.5 (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be

$$\Delta \text{PE}_g = mgh, \tag{7.27}$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mgh &= (0.500 \text{ kg})\left(9.80 \text{ m/s}^2\right)(1.00 \text{ m}) \\ &= 4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}. \end{aligned} \tag{7.28}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta \text{PE}_g = mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See **Figure 7.6**.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

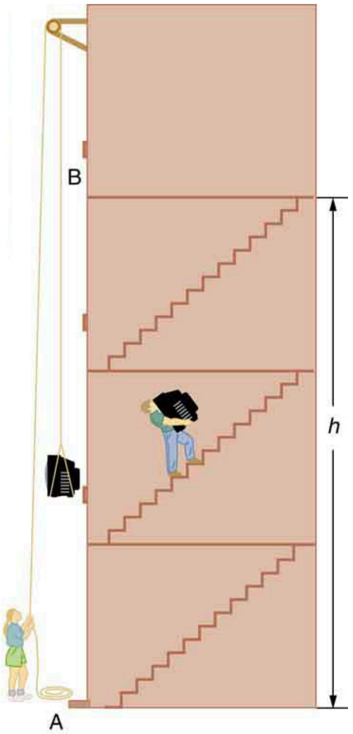

Figure 7.6 The change in gravitational potential energy $(\Delta\text{PE}_\text{g})$ between points A and B is independent of the path. $\Delta\text{PE}_\text{g} = mgh$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example 7.6 The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = Fd\cos\theta = -Fd, \tag{7.29}$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\cos\theta = \cos 180^\circ = -1)$. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$\text{KE} = -\Delta\text{PE}_\text{g} = -mgh, \tag{7.30}$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\text{KE} = mgh. \tag{7.31}$$

Combining this equation with the expression for W gives

$$-Fd = mgh. \tag{7.32}$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -\frac{(60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(-3.00\text{ m})}{5.00\times10^{-3}\text{ m}} = 3.53\times10^{5}\text{ N}. \tag{7.33}$$

Discussion

Such a large force (500 times more than the person’s weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See **Figure 7.7**.)

Figure 7.7 The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example 7.7 Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in **Figure 7.8** if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

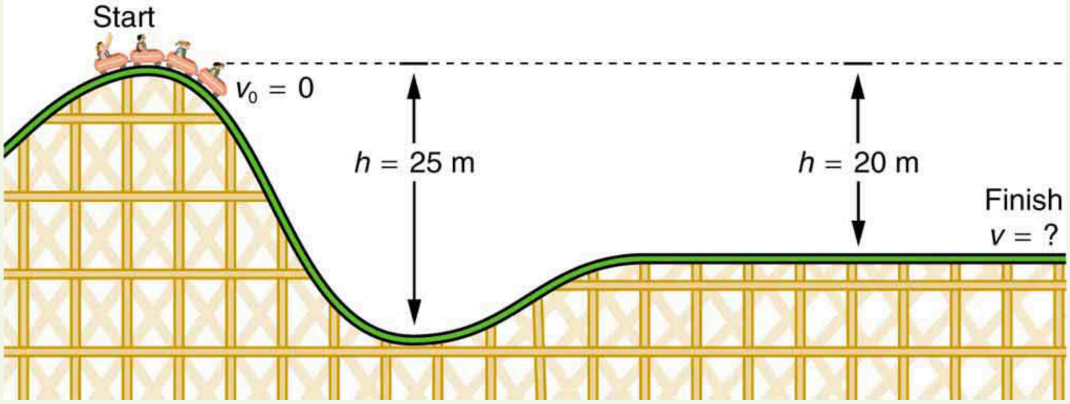

Figure 7.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system’s gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all $\Delta\mathrm{PE_g}$ is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta\mathrm{PE_g} = \Delta\mathrm{KE}$. Using the equations for

$\mathrm{PE_g}$ and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta\mathrm{KE} = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta\mathrm{PE_g} = mgh$. Since h is negative in this case, we will rewrite this as $\Delta\mathrm{PE_g} = -mg \mid h \mid$ to show the minus sign clearly. Thus,

$$-\Delta\mathrm{PE_g} = \Delta\mathrm{KE} \tag{7.34}$$

becomes

$$mg \mid h \mid \ = \frac{1}{2}mv^2. \tag{7.35}$$

Solving for v, we find that mass cancels and that

$$v = \sqrt{2g \mid h \mid}. \tag{7.36}$$

Substituting known values,

$$\begin{aligned} v &= \sqrt{2\left(9.80\ \mathrm{m/s^2}\right)(20.0\ \mathrm{m})} \\ &= 19.8\ \mathrm{m/s}. \end{aligned} \tag{7.37}$$

Solution for (b)

Again $-\Delta\mathrm{PE_g} = \Delta\mathrm{KE}$. In this case there is initial kinetic energy, so $\Delta\mathrm{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$. Thus,

$$mg \mid h \mid \ = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2. \tag{7.38}$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg \mid h \mid \ + \frac{1}{2}mv_0{}^2. \tag{7.39}$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g \mid h \mid \ + v_0{}^2}. \tag{7.40}$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0{}^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m}) + (5.00\ \mathrm{m/s})^2} \\ &= 20.4\ \mathrm{m/s}. \end{aligned} \tag{7.41}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in **Falling Objects** that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see **Figure 7.9**).

Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

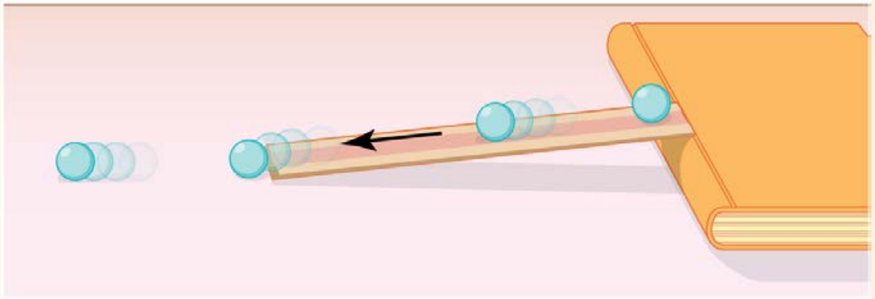

Figure 7.9 A marble rolls down a ruler, and its speed on the level surface is measured.

7.4 Conservative Forces and Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces leads to conservation of mechanical energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **(S.P. 1.4, 2.1, 2.2)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **5.B.1.1** The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. **(S.P. 1.4, 2.2)**
- **5.B.1.2** The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. **(S.P. 1.5)**
- **5.B.3.1** The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. **(S.P. 2.2, 6.4, 7.2)**
- **5.B.3.2** The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. **(S.P. 1.4, 2.2)**
- **5.B.3.3** The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. **(S.P. 1.4, 2.2)**

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Real World Connections: Energy of a Bowling Ball

How much energy does a bowling ball have? (Just think about it for a minute.)

If you are thinking that you need more information, you're right. If we can measure the ball's velocity, then determining its kinetic energy is simple. Note that this does require defining a reference frame in which to measure the velocity. Determining the ball's potential energy also requires more information. You need to know its height above the ground, which requires a reference frame of the ground. Without the ground—in other words, Earth—the ball does not classically have potential energy. Potential energy comes from the interaction between the ball and the ground. Another way of thinking about this is to compare the ball's potential energy on Earth and on the Moon. A bowling ball a certain height above Earth is going to have more potential energy than the same bowling ball the same height above the surface of the Moon, because Earth has greater mass than the Moon and therefore exerts more gravity on the ball. Thus, potential energy requires a system of at least two objects, or an object with an internal structure of at least two parts.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring (PE_s). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in **Elasticity: Stress and Strain**, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See **Figure 7.10**.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_\mathrm{s} = Fd = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$. Alternatively, we noted in **Kinetic Energy and the Work-Energy Theorem** that the area under a graph of F vs. x is the work done by the force. In **Figure 7.10**(c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, PE_s , to be

$$\mathrm{PE}_\mathrm{s} = \frac{1}{2}kx^2, \tag{7.42}$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x . The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

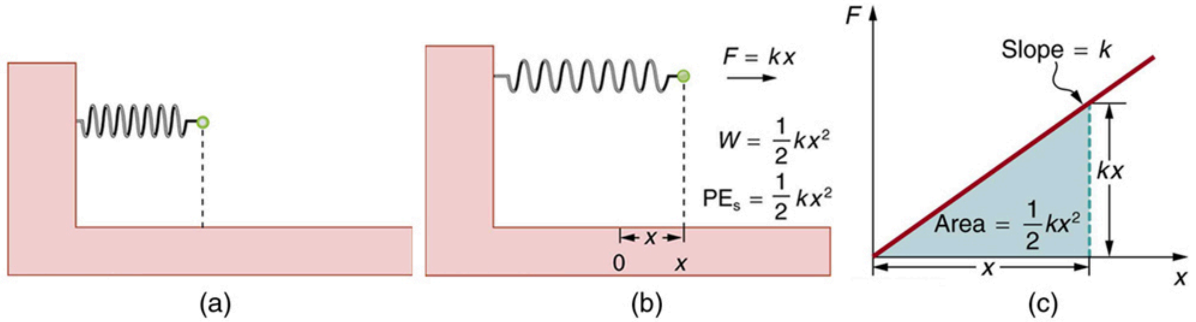

Figure 7.10 (a) An undeformed spring has no PE_s stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F = kx$, and the work done to stretch (or compress) it is $\frac{1}{2}kx^2$. Because the force is conservative, this work is stored as potential energy (PE_s) in the spring, and it can be fully recovered. (c) A graph of F vs. x has a slope of k , and the area under the graph is $\frac{1}{2}kx^2$. Thus the work done or potential energy stored is $\frac{1}{2}kx^2$.

The equation $\mathrm{PE}_\mathrm{s} = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $\text{PE}_\text{s} = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in **Figure 7.11** for a guitar string.

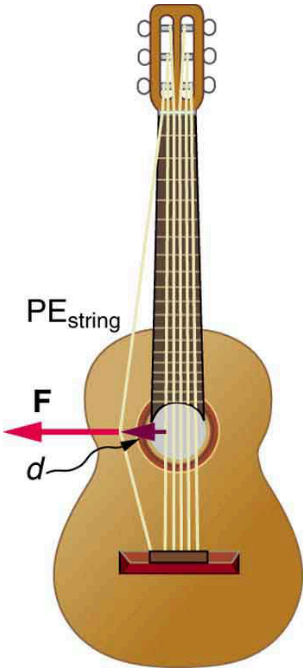

Figure 7.11 Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_\text{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2 = \Delta\text{KE}. \tag{7.43}$$

If only conservative forces act, then

$$W_\text{net} = W_\text{c}, \tag{7.44}$$

where W_c is the total work done by all conservative forces. Thus,

$$W_\text{c} = \Delta\text{KE}. \tag{7.45}$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_\text{c} = -\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE} = \Delta\text{KE} \tag{7.46}$$

or

$$\Delta\text{KE} + \Delta\text{PE} = 0. \tag{7.47}$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left.\begin{array}{c} \text{KE} + \text{PE} = \text{constant} \\ \text{or} \\ \text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} \end{array}\right\}\text{(conservative forces only)}, \tag{7.48}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\text{KE} + \text{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE , with the total energy remaining constant.

The internal energy of a system is the sum of the kinetic energies of all of its elements, plus the potential energy due to all of the interactions due to conservative forces between all of the elements.

Real World Connections

Consider a wind-up toy, such as a car. It uses a spring system to store energy. The amount of energy stored depends only on how many times it is wound, not how quickly or slowly the winding happens. Similarly, a dart gun using compressed air stores energy in its internal structure. In this case, the energy stored inside depends only on how many times it is pumped, not how quickly or slowly the pumping is done. The total energy put into the system, whether through winding or pumping, is equal to the total energy conserved in the system (minus any energy loss in the system due to interactions between its parts, such as air leaks in the dart gun). Since the internal energy of the system is conserved, you can calculate the amount of stored energy by measuring the kinetic energy of the system (the moving car or dart) when the potential energy is released.

Example 7.8 Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in **Figure 7.12**. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

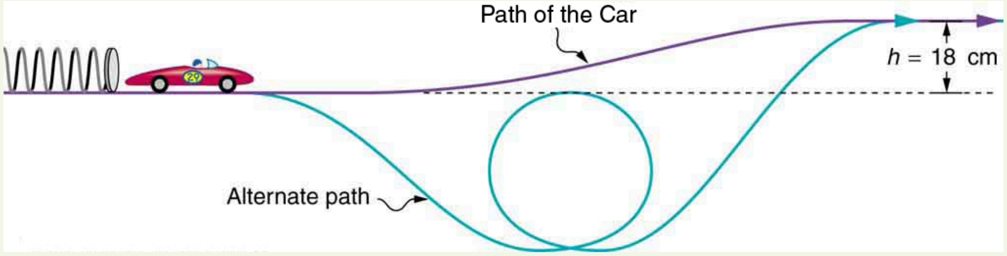

Figure 7.12 A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \tag{7.49}$$

or

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2, \tag{7.50}$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2. \tag{7.51}$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}}x_i \\ &= \sqrt{\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}}(0.0400\ \text{m}) \\ &= 2.00\ \text{m/s}. \end{aligned} \tag{7.52}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_{\mathrm{i}}^{2}=\frac{1}{2}mv_{\mathrm{f}}^{2}+mgh_{\mathrm{f}}. \tag{7.53}$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_{f} and substituting known values gives

$$\begin{aligned} v_{\mathrm{f}} &= \sqrt{\frac{kx_{\mathrm{i}}^{2}}{m}-2gh_{\mathrm{f}}} \\ &= \sqrt{\left(\frac{250.0\ \mathrm{N/m}}{0.100\ \mathrm{kg}}\right)(0.0400\ \mathrm{m})^{2}-2(9.80\ \mathrm{m/s^{2}})(0.180\ \mathrm{m})} \\ &= 0.687\ \mathrm{m/s}. \end{aligned} \tag{7.54}$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Applying the Science Practices: Potential Energy in a Spring

Suppose you are running an experiment in which two 250 g carts connected by a spring (with spring constant 120 N/m) are run into a solid block, and the compression of the spring is measured. In one run of this experiment, the spring was measured to compress from its rest length of 5.0 cm to a minimum length of 2.0 cm. What was the potential energy stored in this system?

Answer

Note that the change in length of the spring is 3.0 cm. Hence we can apply Equation 7.42 to find that the potential energy is $PE = (1/2)(120\ \mathrm{N/m})(0.030\ \mathrm{m})^2 = 0.0541\ \mathrm{J}$.

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in **Example 7.8**. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

PhET Explorations: Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

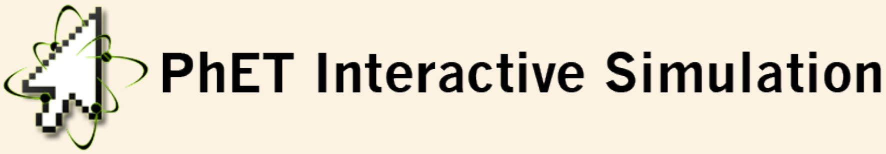

Figure 7.13 Energy Skate Park (http://cnx.org/content/m55076/1.5/energy-skate-park_en.jar)

7.5 Nonconservative Forces

Learning Objectives

By the end of this section, you will be able to:

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.2** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **(S.P. 6.4)**

- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in **Conservative Forces and Potential Energy**. A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in **Figure 7.14**, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

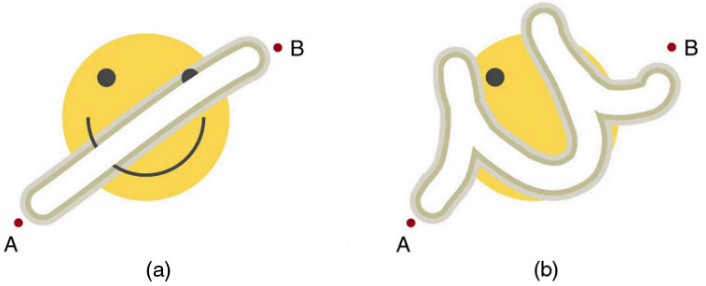

Figure 7.14 The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. **Figure 7.15** compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in **Figure 7.15**(a) first before studying more complicated systems as in **Figure 7.15**(b).

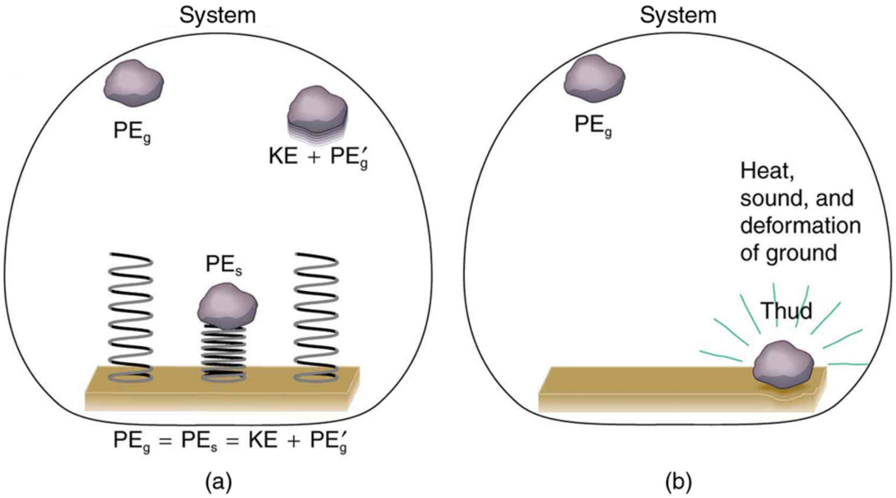

Figure 7.15 Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in **Kinetic Energy and the Work-Energy Theorem**, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{\text{net}} = \Delta\text{KE}$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{\mathrm{net}} = W_{\mathrm{nc}} + W_{\mathrm{c}}, \tag{7.55}$$

so that

$$W_{\mathrm{nc}} + W_{\mathrm{c}} = \Delta\mathrm{KE}, \tag{7.56}$$

where W_{nc} is the total work done by all nonconservative forces and W_{c} is the total work done by all conservative forces.

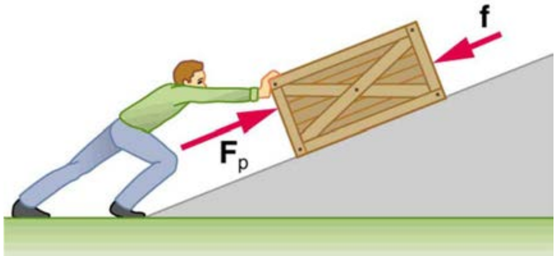

Figure 7.16 A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider **Figure 7.16**, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_{\mathrm{c}} = -\Delta\mathrm{PE}$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{\mathrm{nc}} = \Delta\mathrm{KE} + \Delta\mathrm{PE}. \tag{7.57}$$

This equation means that the total mechanical energy $(\mathrm{KE} + \mathrm{PE})$ changes by exactly the amount of work done by nonconservative forces. In **Figure 7.16**, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{\mathrm{nc}} = \Delta\mathrm{KE} + \Delta\mathrm{PE}$ to obtain

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}. \tag{7.58}$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in **Figure 7.16**. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in **Figure 7.15**(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example 7.9 Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in **Figure 7.17**, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

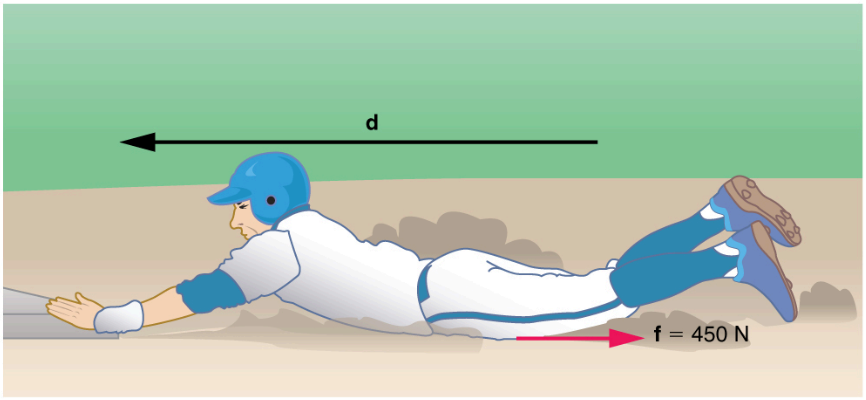

Figure 7.17 The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because $\mathbf{f}$ is in the opposite direction of the motion (that is, $\theta = 180^\circ$, and so $\cos\theta = -1$). Thus $W_{\text{nc}} = -fd$. The equation simplifies to

$$\frac{1}{2}mv_{\text{i}}^2 - fd = 0 \tag{7.59}$$

or

$$fd = \frac{1}{2}mv_{\text{i}}^2. \tag{7.60}$$

This equation can now be solved for the distance d.

Solution

Solving the previous equation for d and substituting known values yields

$$\begin{aligned} d &= \frac{mv_{\text{i}}^2}{2f} \\ &= \frac{(65.0\text{ kg})(6.00\text{ m/s})^2}{(2)(450\text{ N})} \\ &= 2.60\text{ m}. \end{aligned} \tag{7.61}$$

Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example 7.10 Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from **Example 7.9** is running up a hill having a 5.00° incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.

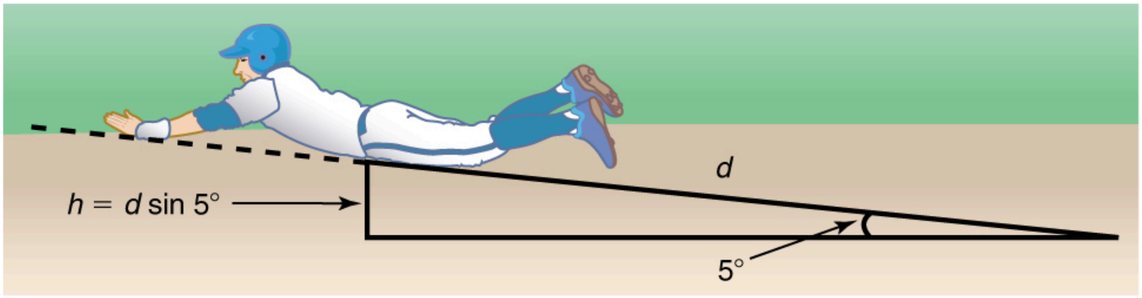

Figure 7.18 The same baseball player slides to a stop on a 5.00° slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d \sin 5.00^\circ$. This is expressed by the equation

$$\text{KE} + \text{PE}_\text{i} + W_\text{nc} = \text{KE}_\text{f} + \text{PE}_\text{f}. \tag{7.62}$$

Solution

The work done by friction is again $W_\text{nc} = -fd$; initially the potential energy is $\text{PE}_\text{i} = mg \cdot 0 = 0$ and the kinetic energy is $\text{KE}_\text{i} = \frac{1}{2}mv_\text{i}^2$; the final energy contributions are $\text{KE}_\text{f} = 0$ for the kinetic energy and $\text{PE}_\text{f} = mgh = mgd \sin \theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_\text{i}^2 + 0 + \left(-fd\right) = 0 + mgd \sin \theta. \tag{7.63}$$

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\left(\frac{1}{2}\right)mv_\text{i}^2}{f + mg \sin \theta} \\ &= \frac{(0.5)(65.0 \text{ kg})(6.00 \text{ m/s})^2}{450 \text{ N}+(65.0 \text{ kg})(9.80 \text{ m/s}^2) \sin (5.00^\circ)} \\ &= 2.31 \text{ m}. \end{aligned} \tag{7.64}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from **Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy**. In addition, you will need a foam cup with a small hole in the side, as shown in **Figure 7.19**. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_k of the cup on the table. The force of friction f on the cup is $\mu_\text{k} N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd . You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

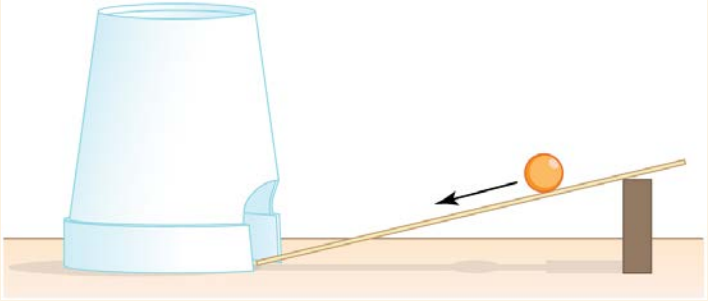

Figure 7.19 Rolling a marble down a ruler into a foam cup.

PhET Explorations: The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

PhET Interactive Simulation

Figure 7.20 The Ramp (http://cnx.org/content/m55047/1.5/the-ramp_en.jar)

7.6 Conservation of Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.C.1.2** The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **(S.P. 6.4)**
- **4.C.2.1** The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **(S.P. 6.4)**
- **4.C.2.2** The student is able to apply the concepts of conservation of energy and the work-energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **(S.P. 1.4, 2.2, 7.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.B.5.4** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). **(S.P. 6.4, 7.2)**
- **5.B.5.5** The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. **(S.P. 2.2, 6.4)**

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of**

conservation of energy can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\text{KE} + \text{PE})$ and energy transferred via work done by nonconservative forces (W_{nc}). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$\text{KE}_\text{i} + \text{PE}_\text{i} + W_{\text{nc}} + \text{OE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} + \text{OE}_\text{f}. \quad (7.65)$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc}, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Real World Connections: Open or Closed System?

Consider whether the following systems are open or closed: a car, a spring-operated dart gun, and the system shown in **Figure 7.15**(a).

A car is not a closed system. You add energy in the form of more gas in the tank (or charging the batteries), and energy is lost due to air resistance and friction.

A spring-operated dart gun is not a closed system. You have to initially compress the spring. Once that has been done, however, the dart gun and dart can be treated as a closed system. All of the energy remains in the system consisting of these two objects.

Figure 7.15(a) is an example of a closed system, once it has been started. All of the energy in the system remains there; none is brought in from outside or leaves.

Table 7.1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}. \tag{7.66}$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}. \tag{7.67}$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c , the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that $\mathrm{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see **Figure 7.21**) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 7.21 Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Table 7.1 Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth’s oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $\left(10^{-2}\text{ g at }0.5\text{ m/s}\right)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** Eff of an energy conversion process is defined as

$$\text{Efficien}\quad(\mathit{Eff}) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}. \tag{7.68}$$

Table 7.2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Table 7.2 Efficiency of the Human Body and Mechanical Devices

Activity/device	Efficiency (%)[1]
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

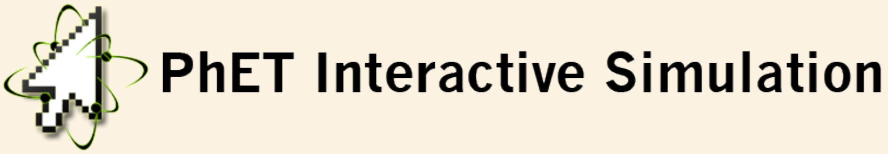

Figure 7.22 Masses and Springs (http://cnx.org/content/m55049/1.4/mass-spring-lab_en.jar)

7.7 Power

Learning Objectives

By the end of this section, you will be able to:

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in **Figure 7.23**.

---

[1] Representative values

Figure 7.23 This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Power

Power is the rate at which work is done.

$$P = \frac{W}{t} \tag{7.69}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1\text{ W} = 1\text{ J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 7.11 Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See **Figure 7.24**.)

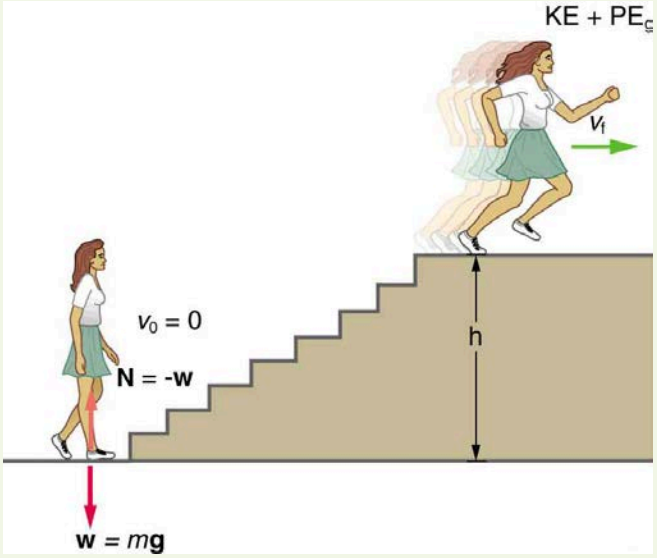

Figure 7.24 When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus, $W = \text{KE}_\text{f} + \text{PE}_\text{g} = \frac{1}{2}mv_\text{f}^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_\text{f}^2 + mgh}{t}. \tag{7.70}$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2 + (60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J} + 1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned} \tag{7.71}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\text{ hp} = 746\text{ W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See **Table 7.3** for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See **Figure 7.25**.)

Figure 7.25 Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Table 7.3 Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt. \tag{7.72}$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\text{kW}\cdot\text{h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 7.12 Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\text{kW}\cdot\text{h}$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $\text{kW}\cdot\text{h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\text{kW}\cdot\text{h}$ is

$$\begin{aligned} E &= Pt = (0.200\ \text{kW})(6.00\ \text{h/d})(30.0\ \text{d}) \\ &= 36.0\ \text{kW}\cdot\text{h}, \end{aligned} \tag{7.73}$$

and the cost is simply given by

$$\text{cost} = (36.0\ \text{kW}\cdot\text{h})(\$0.120\ \text{per kW}\cdot\text{h}) = \$4.32\ \text{per month}. \tag{7.74}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in **Thermodynamics**, the potential for energy to produce useful work has been “degraded” in the energy transformation.

7.8 Work, Energy, and Power in Humans

Learning Objectives

By the end of this section, you will be able to:

- Explain the human body’s consumption of energy when at rest versus when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See **Figure 7.26**.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

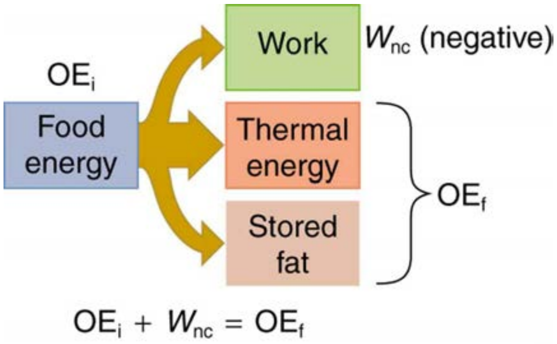

Figure 7.26 Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in **Table 7.4**. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Table 7.4 Basal Metabolic Rates (BMR)

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See **Figure 7.27**.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. **Table 7.5** shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy ($\text{KE} + \text{PE}$) of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as **Example 7.13** illustrates.

Example 7.13 Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table 7.5 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000 \text{ kJ}}{400 \text{ W}} = 2500 \text{ s} = 42 \text{ min}. \tag{7.75}$$

Discussion

If this person uses more energy than he or she consumes, the person’s body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\text{Fat loss} = (1000\text{ kJ})\left(\frac{1.0\text{ g fat}}{39\text{ kJ}}\right) = 26\text{ g}, \tag{7.76}$$

assuming the energy content of fat to be 39 kJ/g.

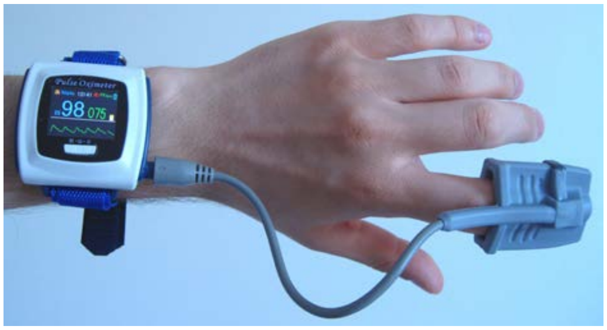

Figure 7.27 A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person’s metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

Table 7.5 Energy and Oxygen Consumption Rates[2] (Power)

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

All bodily functions, from thinking to lifting weights, require energy. (See **Figure 7.28**.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

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[2] for an average 76-kg male

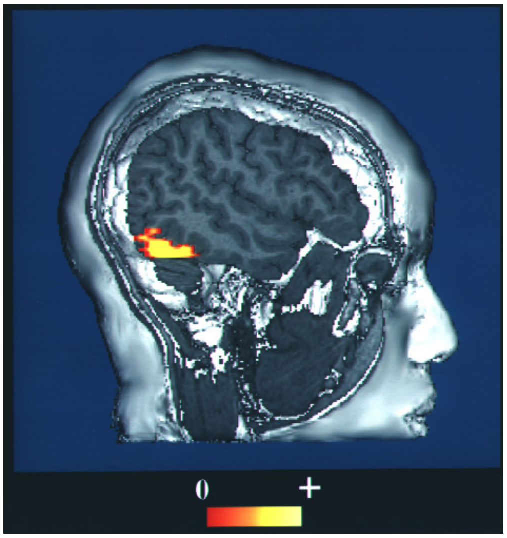

Figure 7.28 This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

7.9 World Energy Use

Learning Objectives

By the end of this section, you will be able to:

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in **Figure 7.29**. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

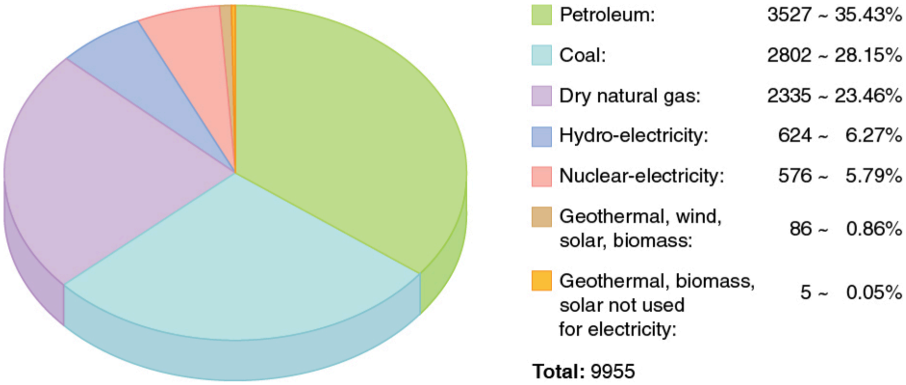

Figure 7.29 World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See **Figure 7.30**.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See **Figure 7.31**.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO_2. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.

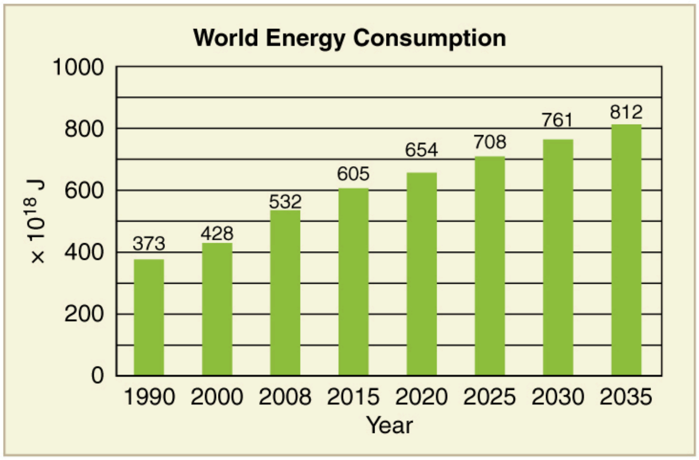

Figure 7.30 Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2011)

Figure 7.31 Solar cell arrays at a power plant in Steindorf, Germany (credit: Michael Betke, Flickr)

Table 7.6 displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

Table 7.6 Energy Consumption—Selected Countries (2006)

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables	Electricity Use per capita (kWh/yr)	Energy Use per capita (GJ/yr)
Australia	5.4	34%	17%	44%	0%	3%	1%	10000	260
Brazil	9.6	48%	7%	5%	1%	35%	2%	2000	50
China	63	22%	3%	69%	1%	6%		1500	35
Egypt	2.4	50%	41%	1%	0%	6%		990	32
Germany	16	37%	24%	24%	11%	1%	3%	6400	173
India	15	34%	7%	52%	1%	5%		470	13
Indonesia	4.9	51%	26%	16%	0%	2%	3%	420	22
Japan	24	48%	14%	21%	12%	4%	1%	7100	176
New Zealand	0.44	32%	26%	6%	0%	11%	19%	8500	102
Russia	31	19%	53%	16%	5%	6%		5700	202
U.S.	105	40%	23%	22%	8%	3%	1%	12500	340
World	**432**	**39%**	**23%**	**24%**	**6%**	**6%**	**2%**	**2600**	**71**

Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in **Figure 7.32**. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

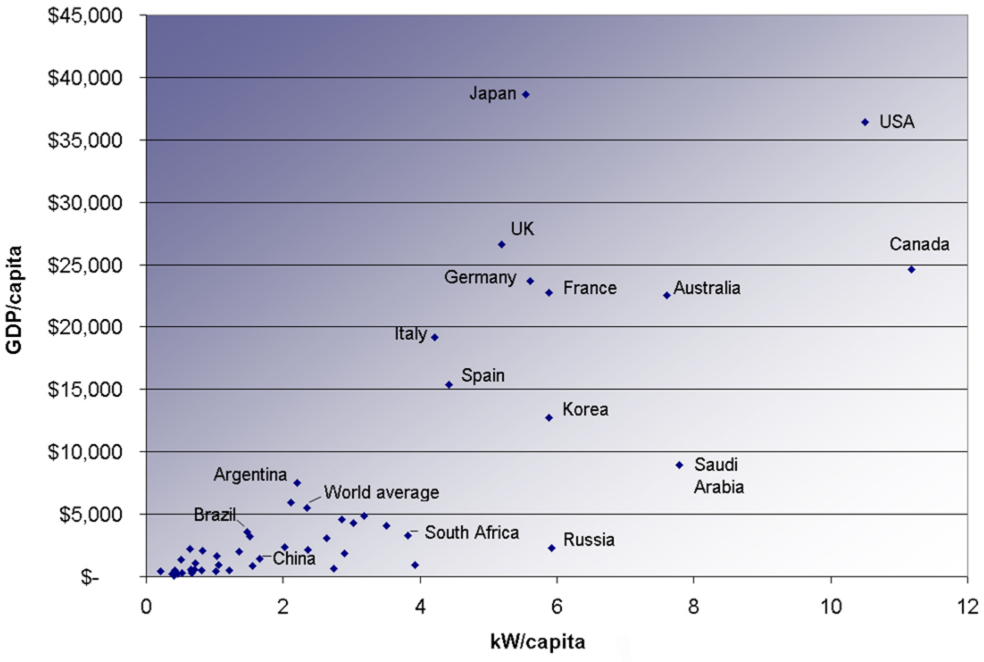

Figure 7.32 Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation. (This will be discussed in more detail in **Thermodynamics**.)

Glossary

basal metabolic rate: the total energy conversion rate of a person at rest

chemical energy: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

conservation of mechanical energy: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

conservative force: a force that does the same work for any given initial and final configuration, regardless of the path followed

efficiency: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

electrical energy: the energy carried by a flow of charge

energy: the ability to do work

fossil fuels: oil, natural gas, and coal

friction: the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

gravitational potential energy: the energy an object has due to its position in a gravitational field

horsepower: an older non-SI unit of power, with $1 \text{ hp} = 746 \text{ W}$

joule: SI unit of work and energy, equal to one newton-meter

kilowatt-hour: $(\text{kW} \cdot \text{h})$ unit used primarily for electrical energy provided by electric utility companies

kinetic energy: the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

law of conservation of energy: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

mechanical energy: the sum of kinetic energy and potential energy

metabolic rate: the rate at which the body uses food energy to sustain life and to do different activities

net work: work done by the net force, or vector sum of all the forces, acting on an object

nonconservative force: a force whose work depends on the path followed between the given initial and final configurations

nuclear energy: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

potential energy: energy due to position, shape, or configuration

potential energy of a spring: the stored energy of a spring as a function of its displacement; when Hooke’s law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

power: the rate at which work is done

radiant energy: the energy carried by electromagnetic waves

renewable forms of energy: those sources that cannot be used up, such as water, wind, solar, and biomass

thermal energy: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

useful work: work done on an external system

watt: (W) SI unit of power, with $1 \text{ W} = 1 \text{ J/s}$

work: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

work-energy theorem: the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

Section Summary

7.1 Work: The Scientific Definition

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd \cos \theta .$$

- The SI unit for work and energy is the joule (J), where $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

7.2 Kinetic Energy and the Work-Energy Theorem

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$.

7.3 Gravitational Potential Energy

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta\text{PE}_\text{g} = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth’s surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta\text{KE}= -\Delta\text{PE}_\text{g}$.

7.4 Conservative Forces and Potential Energy

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\text{PE}_\text{s} = \frac{1}{2}kx^2$, where k is the spring’s force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\left.\begin{array}{c}\mathrm{KE} + \mathrm{PE} = \text{constant} \\ \text{or} \\ \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}\end{array}\right\}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

7.5 Nonconservative Forces

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{\mathrm{nc}} = \Delta\mathrm{KE} + \Delta\mathrm{PE}$ or, equivalently, $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

7.6 Conservation of Energy

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $\mathit{Eff} = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

7.7 Power

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t , $P = W/t$.
- The SI unit for power is the watt (W), where $1\ \mathrm{W} = 1\ \mathrm{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\ \mathrm{hp} = 746\ \mathrm{W}$.

7.8 Work, Energy, and Power in Humans

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

7.9 World Energy Use

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Conceptual Questions

7.1 Work: The Scientific Definition

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

7.2 Kinetic Energy and the Work-Energy Theorem

4. The person in **Figure 7.33** does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

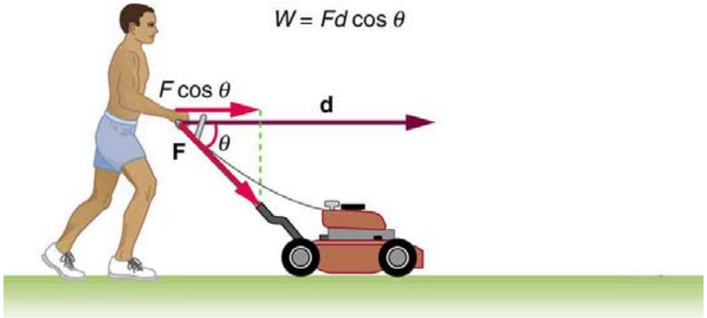

Figure 7.33

5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

6. When solving for speed in **Example 7.4**, we kept only the positive root. Why?

7.3 Gravitational Potential Energy

7. In **Example 7.7**, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s uphill instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial. Explain in terms of conservation of energy.

8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

7.4 Conservative Forces and Potential Energy

9. What is a conservative force?

10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

12. What is the relationship of potential energy to conservative force?

7.6 Conservation of Energy

13. Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See **Figure 7.34**.)

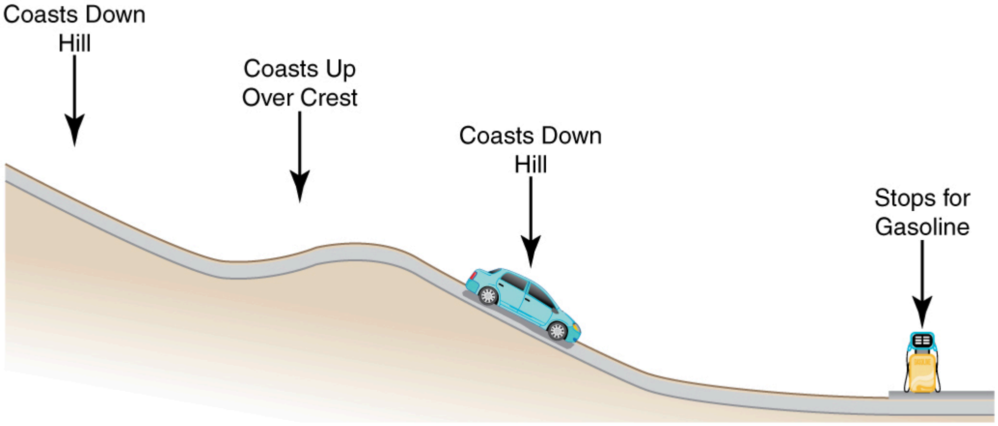

Figure 7.34 A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

15. Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

16. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

17. List the energy conversions that occur when riding a bicycle.

7.7 Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

7.8 Work, Energy, and Power in Humans

21. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

22. Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

23. Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

24. Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

7.9 World Energy Use

25. What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

26. If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

Problems & Exercises

7.1 Work: The Scientific Definition

1. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

2. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

3. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

4. Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See **Table 7.1** for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

5. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See **Figure 7.35**.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.

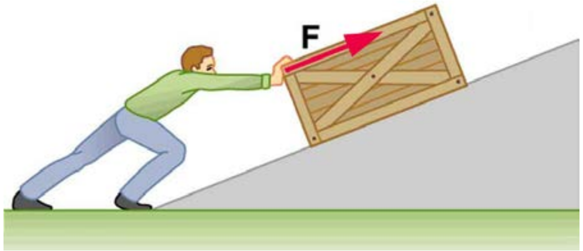

Figure 7.35 A man pushes a crate up a ramp.

6. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in **Figure 7.36**? Assume no friction acts on the wagon.

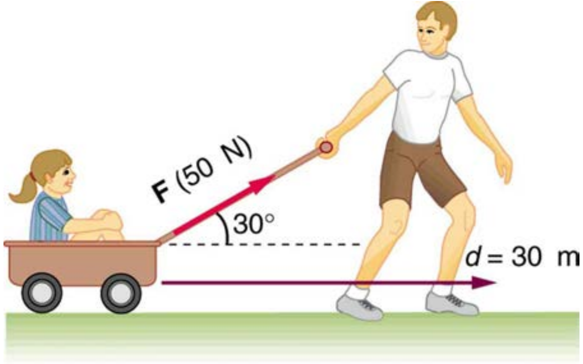

Figure 7.36 The boy does work on the system of the wagon and the child when he pulls them as shown.

7. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

8. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in **Figure 7.37**. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

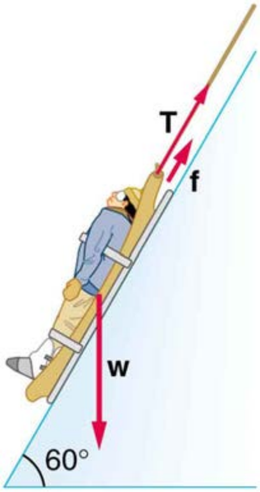

Figure 7.37 A rescue sled and victim are lowered down a steep slope.

7.2 Kinetic Energy and the Work-Energy Theorem

9. Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

10. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

11. Confirm the value given for the kinetic energy of an aircraft carrier in **Table 7.1**. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

12. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

13. A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

14. Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

15. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

7.3 Gravitational Potential Energy

16. A hydroelectric power facility (see **Figure 7.38**) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0 km^3 ($\text{mass} = 5.00\times10^{13}\text{ kg})$, given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.

Figure 7.38 Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

17. (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9\text{ kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

18. Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

19. In **Example 7.7**, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta\text{PE} >> \text{KE}_\text{i}$. Confirm this statement by taking the ratio of ΔPE to KE_i . (Note that mass cancels.)

20. A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in **Figure 7.39**. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

Figure 7.39 A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

21. In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30^o slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

7.4 Conservative Forces and Potential Energy

22. A $5.00\times10^5\text{-kg}$ subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

23. A pogo stick has a spring with a force constant of $2.50\times10^4\text{ N/m}$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Energy**.

7.5 Nonconservative Forces

24. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in **Figure 7.40**. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

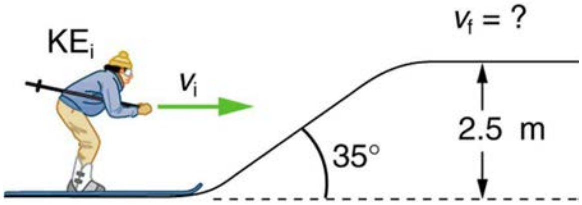

Figure 7.40 The skier's initial kinetic energy is partially used in coasting to the top of a rise.

25. (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5^{o} above the horizontal?

7.6 Conservation of Energy

26. Using values from **Table 7.1**, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

27. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

28. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from **Table 7.1**)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

29. (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from **Table 7.1**. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

7.7 Power

30. The Crab Nebula (see **Figure 7.41**) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from **Table 7.3**, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

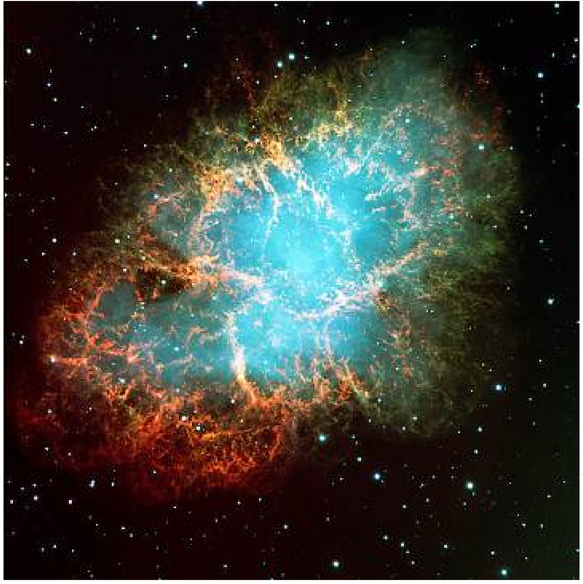

Figure 7.41 Crab Nebula (credit: ESO, via Wikimedia Commons)

31. Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from **Table 7.3**: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

32. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

33. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

34. A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $\text{kW} \cdot \text{h}$?

35. (a) What is the average power consumption in watts of an appliance that uses $5.00 \text{ kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

36. (a) What is the average useful power output of a person who does $6.00\times10^{6} \text{ J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

37. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

38. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

39. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

40. (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00\times10^{4}\text{ J}$ run a pocket calculator that consumes energy at the rate of $1.00\times10^{-3}\text{ W}$?

41. (a) How long would it take a 1.50×10^{5}-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

42. Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Energy**.

43. (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00\times10^{26}\text{ W.}$) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30 kW/m^2 reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05\times10^{20}\text{ J})?$ Australia's energy needs $(5.4\times10^{18}\text{ J})?$ China's energy needs $(6.3\times10^{19}\text{ J})?$ (These energy consumption values are from 2006.)

7.8 Work, Energy, and Power in Humans

44. (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

45. (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

46. Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Figure 7.42 Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

47. (a) What is the efficiency of an out-of-condition professor who does $2.10\times10^{5}\text{ J}$ of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

48. Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from **Table 7.5** for the energy consumption rates of these activities.

49. Using data from **Table 7.5**, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

50. What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See **Table 7.5**.)

51. Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

52. Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

53. Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

54. (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

55. Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see **Figure 7.43**). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from **Table 7.2**, calculate the food energy in kilojoules he metabolized during the flight.

Figure 7.43 The Daedalus 88 in flight. (credit: NASA photo by Beasley)

56. The swimmer shown in **Figure 7.44** exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

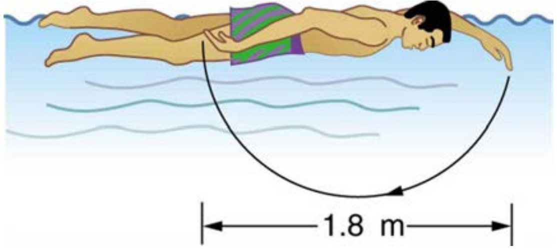

Figure 7.44

57. Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

58. The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7\times10^9\ \text{kg}$. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see **Figure 7.45**), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

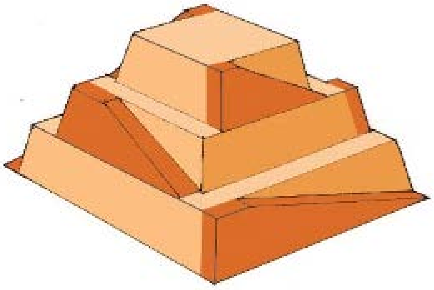

Figure 7.45 Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

59. (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

7.9 World Energy Use

60. Integrated Concepts

(a) Calculate the force the woman in **Figure 7.46** exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in **Work, Energy, and Power in Humans**.

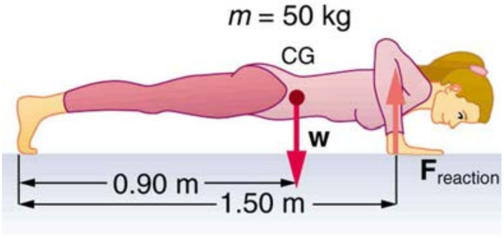

Figure 7.46 Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

61. Integrated Concepts

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

62. Integrated Concepts

The 70.0-kg swimmer in **Figure 7.44** starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

63. Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

64. Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of 0.800 m/s^2 against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

65. Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

66. Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

67. Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

68. Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

69. Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

Test Prep for AP® Courses

7.1 Work: The Scientific Definition

1. Given Table 7.7 about how much force does the rocket engine exert on the 3.0-kg payload?

Table 7.7

Distance traveled with rocket engine firing (m)	Payload final velocity (m/s)
500	310
490	300
1020	450
505	312

a. 150 N
b. 300 N
c. 450 N
d. 600 N

2. You have a cart track, a cart, several masses, and a position-sensing pulley. Design an experiment to examine how the force exerted on the cart does work as it moves through a distance.

3. Look at Figure 7.10(c). You compress a spring by *x*, and then release it. Next you compress the spring by 2*x*. How much more work did you do the second time than the first?
a. Half as much
b. The same
c. Twice as much
d. Four times as much

4. You have a cart track, two carts, several masses, a position-sensing pulley, and a piece of carpet (a rough surface) that will fit over the track. Design an experiment to examine how the force exerted on the cart does work as the cart moves through a distance.

5. A crane is lifting construction materials from the ground to an elevation of 60 m. Over the first 10 m, the motor linearly increases the force it exerts from 0 to 10 kN. It exerts that constant force for the next 40 m, and then winds down to 0 N again over the last 10 m, as shown in the figure. What is the total work done on the construction materials?

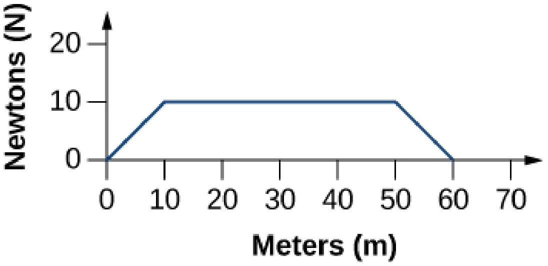

Figure 7.47

a. 500 kJ
b. 600 kJ
c. 300 kJ
d. 18 MJ

7.2 Kinetic Energy and the Work-Energy Theorem

6. A toy car is going around a loop-the-loop. Gravity ____ the kinetic energy on the upward side of the loop, ____ the kinetic energy at the top, and ____ the kinetic energy on the downward side of the loop.
a. increases, decreases, has no effect on
b. decreases, has no effect on, increases
c. increases, has no effect on, decreases
d. decreases, increases, has no effect on

7. A roller coaster is set up with a track in the form of a perfect cosine. Describe and graph what happens to the kinetic energy of a cart as it goes through the first full period of the track.

8. If wind is blowing horizontally toward a car with an angle of 30 degrees from the direction of travel, the kinetic energy will ____. If the wind is blowing at a car at 135 degrees from the direction of travel, the kinetic energy will ____.
a. increase, increase
b. increase, decrease
c. decrease, increase
d. decrease, decrease

9. In what direction relative to the direction of travel can a force act on a car (traveling on level ground), and not change the kinetic energy? Can you give examples of such forces?

10. A 2000-kg airplane is coming in for a landing, with a velocity 5 degrees below the horizontal and a drag force of 40 kN acting directly rearward. Kinetic energy will ____ due to the net force of ____.
a. increase, 20 kN
b. decrease, 40 kN
c. increase, 45 kN
d. decrease, 45 kN

11. You are participating in the Iditarod, and your sled dogs are pulling you across a frozen lake with a force of 1200 N while a 300 N wind is blowing at you at 135 degrees from your direction of travel. What is the net force, and will your kinetic energy increase or decrease?

12. A model drag car is being accelerated along its track from rest by a motor with a force of 75 N, but there is a drag force of 30 N due to the track. What is the kinetic energy after 2 m of travel?
a. 90 J
b. 150 J
c. 210 J
d. 60 J

13. You are launching a 2-kg potato out of a potato cannon. The cannon is 1.5 m long and is aimed 30 degrees above the horizontal. It exerts a 50 N force on the potato. What is the kinetic energy of the potato as it leaves the muzzle of the potato cannon?

14. When the force acting on an object is parallel to the direction of the motion of the center of mass, the mechanical energy ____. When the force acting on an object is antiparallel to the direction of the center of mass, the mechanical energy ____.
a. increases, increases
b. increases, decreases
c. decreases, increases
d. decreases, decreases

15. Describe a system in which the main forces acting are parallel or antiparallel to the center of mass, and justify your answer.

16. A child is pulling two red wagons, with the second one tied to the first by a (non-stretching) rope. Each wagon has a mass of 10 kg. If the child exerts a force of 30 N for 5.0 m, how much has the kinetic energy of the two-wagon system changed?

a. 300 J
b. 150 J
c. 75 J
d. 60 J

17. A child has two red wagons, with the rear one tied to the front by a (non-stretching) rope. If the child pushes on the rear wagon, what happens to the kinetic energy of each of the wagons, and the two-wagon system?

18. Draw a graph of the force parallel to displacement exerted on a stunt motorcycle going through a loop-the-loop versus the distance traveled around the loop. Explain the net change in energy.

7.3 Gravitational Potential Energy

19. A 1.0 kg baseball is flying at 10 m/s. How much kinetic energy does it have? Potential energy?

a. 10 J, 20 J
b. 50 J, 20 J
c. unknown, 50 J
d. 50 J, unknown

20. A 2.0-kg potato has been launched out of a potato cannon at 9.0 m/s. What is the kinetic energy? If you then learn that it is 4.0 m above the ground, what is the total mechanical energy relative to the ground?

a. 78 J, 3 J
b. 160 J, 81 J
c. 81 J, 160 J
d. 81 J, 3 J

21. You have a 120-g yo-yo that you are swinging at 0.9 m/s. How much energy does it have? How high can it get above the lowest point of the swing without your doing any additional work, on Earth? How high could it get on the Moon, where gravity is 1/6 Earth's?

7.4 Conservative Forces and Potential Energy

22. Two 4.0 kg masses are connected to each other by a spring with a force constant of 25 N/m and a rest length of 1.0 m. If the spring has been compressed to 0.80 m in length and the masses are traveling toward each other at 0.50 m/s (each), what is the total energy in the system?

a. 1.0 J
b. 1.5 J
c. 9.0 J
d. 8.0 J

23. A spring with a force constant of 5000 N/m and a rest length of 3.0 m is used in a catapult. When compressed to 1.0 m, it is used to launch a 50 kg rock. However, there is an error in the release mechanism, so the rock gets launched almost straight up. How high does it go, and how fast is it going when it hits the ground?

24. What information do you need to calculate the kinetic energy and potential energy of a spring? Potential energy due to gravity? How many objects do you need information about for each of these cases?

25. You are loading a toy dart gun, which has two settings, the more powerful with the spring compressed twice as far as the lower setting. If it takes 5.0 J of work to compress the dart gun to the lower setting, how much work does it take for the higher setting?

a. 20 J
b. 10 J
c. 2.5 J
d. 40 J

26. Describe a system you use daily with internal potential energy.

27. Old-fashioned pendulum clocks are powered by masses that need to be wound back to the top of the clock about once a week to counteract energy lost due to friction and to the chimes. One particular clock has three masses: 4.0 kg, 4.0 kg, and 6.0 kg. They can drop 1.3 meters. How much energy does the clock use in a week?

a. 51 J
b. 76 J
c. 127 J
d. 178 J

28. A water tower stores not only water, but (at least part of) the energy to move the water. How much? Make reasonable estimates for how much water is in the tower, and other quantities you need.

29. Old-fashioned pocket watches needed to be wound daily so they wouldn't run down and lose time, due to the friction in the internal components. This required a large number of turns of the winding key, but not much force per turn, and it was possible to overwind and break the watch. How was the energy stored?

a. A small mass raised a long distance
b. A large mass raised a short distance
c. A weak spring deformed a long way
d. A strong spring deformed a short way

30. Some of the very first clocks invented in China were powered by water. Describe how you think this was done.

7.5 Nonconservative Forces

31. You are in a room in a basement with a smooth concrete floor (friction force equals 40 N) and a nice rug (friction force equals 55 N) that is 3 m by 4 m. However, you have to push a very heavy box from one corner of the rug to the opposite corner of the rug. Will you do more work against friction going around the floor or across the rug, and how much extra?

a. Across the rug is 275 J extra
b. Around the floor is 5 J extra
c. Across the rug is 5 J extra
d. Around the floor is 280 J extra

32. In the Appalachians, along the interstate, there are ramps of loose gravel for semis that have had their brakes fail to drive into to stop. Design an experiment to measure how effective this would be.

7.6 Conservation of Energy

33. You do 30 J of work to load a toy dart gun. However, the dart is 10 cm long and feels a frictional force of 10 N while going through the dart gun's barrel. What is the kinetic energy of the fired dart?

a. 30 J
b. 29 J
c. 28 J
d. 27 J

34. When an object is lifted by a crane, it begins and ends its motion at rest. The same is true of an object pushed across a rough surface. Explain why this happens. What are the differences between these systems?

35. A child has two red wagons, with the rear one tied to the front by a stretchy rope (a spring). If the child pulls on the front wagon, the ____ increases.

a. kinetic energy of the wagons
b. potential energy stored in the spring
c. both A and B
d. not enough information

36. A child has two red wagons, with the rear one tied to the front by a stretchy rope (a spring). If the child pulls on the front wagon, the energy stored in the system increases. How do the relative amounts of potential and kinetic energy in this system change over time?

37. Which of the following are closed systems?
a. Earth
b. a car
c. a frictionless pendulum
d. a mass on a spring in a vacuum

38. Describe a real-world example of a closed system.

39. A 5.0-kg rock falls off of a 10 m cliff. If air resistance exerts a force of 10 N, what is the kinetic energy when the rock hits the ground?
a. 400 J
b. 12.6 m/s
c. 100 J
d. 500 J

40. Hydroelectricity is generated by storing water behind a dam, and then letting some of it run through generators in the dam to turn them. If the system is the water, what is the environment that is doing work on it? If a dam has water 100 m deep behind it, how much energy was generated if 10,000 kg of water exited the dam at 2.0 m/s?

41. Before railroads were invented, goods often traveled along canals, with mules pulling barges from the bank. If a mule is exerting a 1200 N force for 10 km, and the rope connecting the mule to the barge is at a 20 degree angle from the direction of travel, how much work did the mule do on the barge?
a. 12 MJ
b. 11 MJ
c. 4.1 MJ
d. 6 MJ

42. Describe an instance today in which you did work, by the scientific definition. Then calculate how much work you did in that instance, showing your work.