8 LINEAR MOMENTUM AND COLLISIONS

Figure 8.1 Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

Chapter Outline

Connection for AP® courses

In this chapter, you will learn about the concept of momentum and the relationship between momentum and force (both vector quantities) applied over a time interval. Have you ever considered why a glass dropped on a tile floor will often break, but a glass dropped on carpet will often remain intact? Both involve changes in momentum, but the actual collision with the floor is different in each case, just as an automobile collision without the benefit of an airbag can have a significantly different outcome than one with an airbag.

You will learn that the interaction of objects (like a glass and the floor or two automobiles) results in forces, which in turn result in changes in the momentum of each object. At the same time, you will see how the law of momentum conservation can be applied to a system to help determine the outcome of a collision.

The content in this chapter supports:

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.D A force exerted on an object can change the momentum of the object.

Essential Knowledge 3.D.2 The change in momentum of an object occurs over a time interval.

Big Idea 4: Interactions between systems can result in changes in those systems.

Enduring Understanding 4.B Interactions with other objects or systems can change the total linear momentum of a system.

Essential Knowledge 4.B.1 The change in linear momentum for a constant-mass system is the product of the mass of the system and the change in velocity of the center of mass.

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved.

Essential Knowledge 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

Essential Knowledge 5.D.2 In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

8.1 Linear Momentum and Force

Learning Objectives

By the end of this section, you will be able to:

- Define linear momentum.
- Explain the relationship between linear momentum and force.
- State Newton's second law of motion in terms of linear momentum.
- Calculate linear momentum given mass and velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.D.1.1** The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force. **(S.P. 4.1)**

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}. \tag{8.1}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}. \tag{8.2}$$

Example 8.1 Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv \tag{8.3}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg} \cdot \text{m/s} \tag{8.4}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410 \text{ kg})(25.0 \text{ m/s}) = 10.3 \text{ kg} \cdot \text{m/s} \tag{8.5}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9. \quad (8.6)$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}, \quad (8.7)$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} \quad (8.8)$$

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}). \quad (8.9)$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}. \quad (8.10)$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}. \quad (8.11)$$

Because $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \quad (8.12)$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 8.2 Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}. \tag{8.13}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_{\text{f}} - v_{\text{i}}). \tag{8.14}$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_{\text{f}} - v_{\text{i}}) \\ &= (0.057\text{ kg})(58\text{ m/s} - 0\text{ m/s}) \\ &= 3.306\text{ kg}\cdot\text{m/s} \approx 3.3\text{ kg}\cdot\text{m/s} \end{aligned} \tag{8.15}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306\text{ kg}\cdot\text{m/s}}{5.0\times10^{-3}\text{ s}} \\ &= 661\text{ N} \approx 660\text{ N}, \end{aligned} \tag{8.16}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = ma$, but one additional step would be required compared with the strategy used in this example.

Making Connections: Illustrative Example

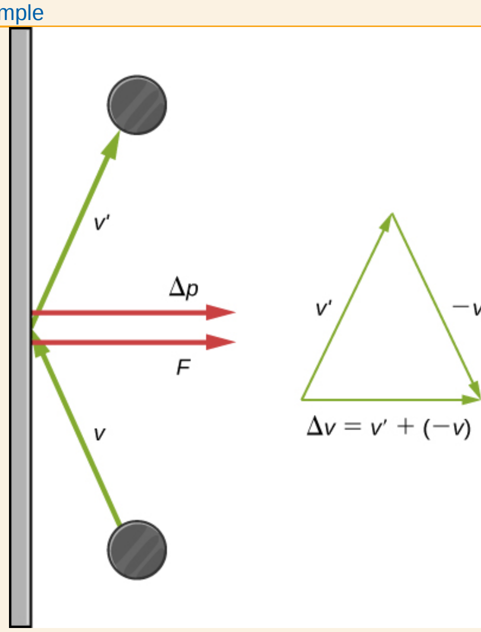


Figure 8.2 A puck has an elastic, glancing collision with the edge of an air hockey table.

In **Figure 8.2**, a puck is shown colliding with the edge of an air hockey table at a glancing angle. During the collision, the edge of the table exerts a force **F** on the puck, and the velocity of the puck changes as a result of the collision. The change in momentum is found by the equation:

$$\Delta \mathrm{p} = m\Delta \mathrm{v} = m\mathrm{v}' - m\mathrm{v} = m(\mathrm{v}' + (-\mathrm{v})) \quad (8.17)$$

As shown, the direction of the change in velocity is the same as the direction of the change in momentum, which in turn is in the same direction as the force exerted by the edge of the table. Note that there is only a horizontal change in velocity. There is no difference in the vertical components of the initial and final velocity vectors; therefore, there is no vertical component to the change in velocity vector or the change in momentum vector. This is consistent with the fact that the force exerted by the edge of the table is purely in the horizontal direction.

8.2 Impulse

Learning Objectives

By the end of this section, you will be able to:

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The information presented in this section supports the following AP® learning objectives and science practices:

- **3.D.2.1** The student is able to justify the selection of routines for the calculation of the relationships between changes in momentum of an object, average force, impulse, and time of interaction. **(S.P. 2.1)**
- **3.D.2.2** The student is able to predict the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **(S.P. 6.4)**
- **3.D.2.3** The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **(S.P. 5.1)**
- **3.D.2.4** The student is able to design a plan for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time. **(S.P. 4.1)**
- **4.B.2.1** The student is able to apply mathematical routines to calculate the change in momentum of a system by analyzing the average force exerted over a certain time on the system. **(S.P. 2.2)**
- **4.B.2.2** The student is able to perform analysis on data presented as a force-time graph and predict the change in momentum of a system. **(S.P. 5.1)**

The effect of a force on an object depends on how long it acts, as well as how great the force is. In **Example 8.1**, a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta \mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \dfrac{\Delta \mathbf{p}}{\Delta t}$ to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}}\Delta t, \quad (8.18)$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}}\Delta t \quad (8.19)$$

The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Making Connections: Illustrations of Force Exerted

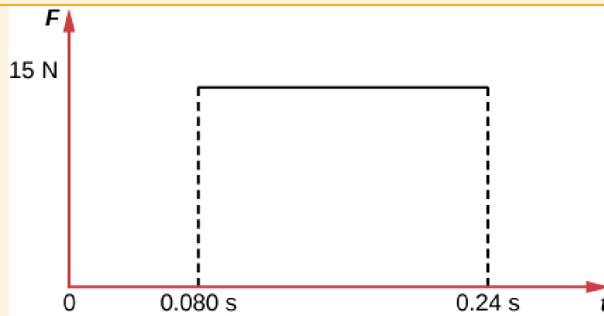


Figure 8.3 This is a graph showing the force exerted by a fixed barrier on a block versus time.

A 1.2-kg block slides across a horizontal, frictionless surface with a constant speed of 3.0 m/s before striking a fixed barrier and coming to a stop. In **Figure 8.3**, the force exerted by the barrier is assumed to be a constant 15 N during the 0.24-s collision. The impulse can be calculated using the area under the curve.

$$\Delta p = F\Delta t = (15\ \ \text{N})(0.24\ \ \text{s}) = \ \ 3.6\ \ \text{kg}\bullet\text{m/s} \tag{8.20}$$

Note that the initial momentum of the block is:

$$p_{initial} = mv_{initial} = (1.2\ \ \text{kg})(-3.0\ \ \text{m/s}) = \ \ -3.6\ \text{kg}\bullet\text{m/s} \tag{8.21}$$

We are assuming that the initial velocity is −3.0 m/s. We have established that the force exerted by the barrier is in the positive direction, so the initial velocity of the block must be in the negative direction. Since the final momentum of the block is zero, the impulse is equal to the change in momentum of the block.

Suppose that, instead of striking a fixed barrier, the block is instead stopped by a spring.Consider the force exerted by the spring over the time interval from the beginning of the collision until the block comes to rest.

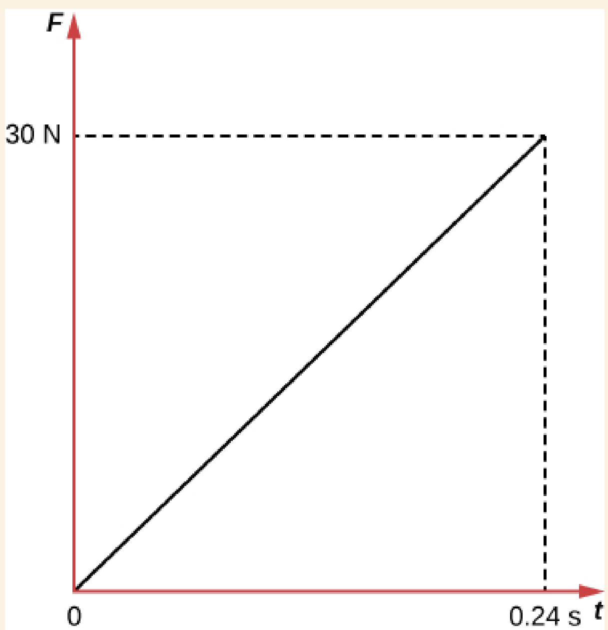


Figure 8.4 This is a graph showing the force exerted by a spring on a block versus time.

In this case, the impulse can be calculated again using the area under the curve (the area of a triangle):

$$\mathit{\Delta} p = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(0.24\ \ \text{s})(30\ \ \text{N}) = \ \ 3.6\ \ \text{kg}\bullet\text{m/s} \tag{8.22}$$

Again, this is equal to the difference between the initial and final momentum of the block, so the impulse is equal to the change in momentum.

Example 8.3 Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the x-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y-axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the $+x$ direction. Therefore the wall exerts a force on the ball in the $-x$ direction. The second ball continues with the same momentum component in the y direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x-axis and y-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{\text{xi}} = mu;\ p_{\text{yi}} = 0 \tag{8.23}$$

$$p_{\text{xf}} = -mu;\ p_{\text{yf}} = 0 \tag{8.24}$$

Impulse is the change in momentum vector. Therefore the x-component of impulse is equal to $-2mu$ and the y-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{\text{xi}} = mu \cos 30^\circ;\ p_{\text{yi}} = -mu \sin 30^\circ \tag{8.25}$$

$$p_{\text{xf}} = -mu \cos 30^\circ;\ p_{\text{yf}} = -mu \sin 30^\circ \tag{8.26}$$

It should be noted here that while p_{x} changes sign after the collision, p_{y} does not. Therefore the x-component of impulse is equal to $-2mu \cos 30^\circ$ and the y-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu \cos 30^\circ} = \frac{2}{\sqrt{3}} = 1.155. \tag{8.27}$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x-direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt. *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. **Figure 8.5** shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2. Thus the impulses and their effects are the same for both the actual and effective forces.

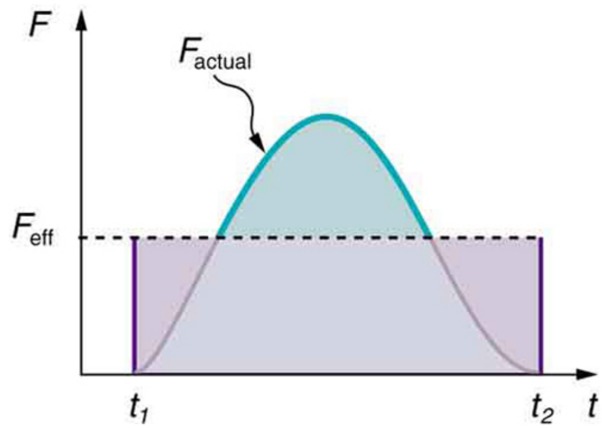


Figure 8.5 A graph of force versus time with time along the x -axis and force along the y -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

Making Connections: Baseball

In most real-life collisions, the forces acting on an object are not constant. For example, when a bat strikes a baseball, the force is very small at the beginning of the collision since only a small portion of the ball is initially in contact with the bat. As the collision continues, the ball deforms so that a greater fraction of the ball is in contact with the bat, resulting in a greater force. As the ball begins to leave the bat, the force drops to zero, much like the force curve in **Figure 8.5**. Although the changing force is difficult to precisely calculate at each instant, the average force can be estimated very well in most cases.

Suppose that a 150-g baseball experiences an average force of 480 N in a direction opposite the initial 32 m/s speed of the baseball over a time interval of 0.017 s. What is the final velocity of the baseball after the collision?

$$\Delta p = F\Delta t = (480)(0.017) = 8.16 \text{ kg}\bullet\text{m/s} \tag{8.28}$$

$$mv_f - mv_i = 8.16 \text{ kg} \bullet \text{m/s} \tag{8.29}$$

$$(0.150 \text{ kg})v_f - (0.150 \text{ kg})(-32 \text{ m/s}) = 8.16 \text{ kg} \bullet \text{m/s} \tag{8.30}$$

$$v_f = 22 \text{ m/s} \tag{8.31}$$

Note in the above example that the initial velocity of the baseball prior to the collision is negative, consistent with the assumption we initially made that the force exerted by the bat is positive and in the direction opposite the initial velocity of the baseball. In this case, even though the force acting on the baseball varies with time, the average force is a good approximation of the effective force acting on the ball for the purposes of calculating the impulse and the change in momentum.

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Making Connections: Constant Force and Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

Applying the Science Practices: Verifying the Relationship between Force and Change in Linear Momentum

Design an experiment in order to experimentally verify the relationship between the impulse of a force and change in linear momentum. For simplicity, it would be best to ensure that frictional forces are very small or zero in your experiment so that the effect of friction can be neglected. As you design your experiment, consider the following:

- Would it be easier to analyze a one-dimensional collision or a two-dimensional collision?
- How will you measure the force?
- Should you have two objects in motion or one object bouncing off a rigid surface?
- How will you measure the duration of the collision?
- How will you measure the initial and final velocities of the object(s)?

- Would it be easier to analyze an elastic or inelastic collision?
- Should you verify the relationship mathematically or graphically?

8.3 Conservation of Momentum

Learning Objectives

By the end of this section, you will be able to:

- Describe the law of conservation of linear momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the law of conservation of momentum as it relates to atomic and subatomic particles.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.4** The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome. **(S.P. 4.2, 5.1, 5.3, 6.4)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.3.1** The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force). **(S.P. 6.4)**

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in **Impulse** and **Linear Momentum and Force**, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in **Figure 8.6**. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

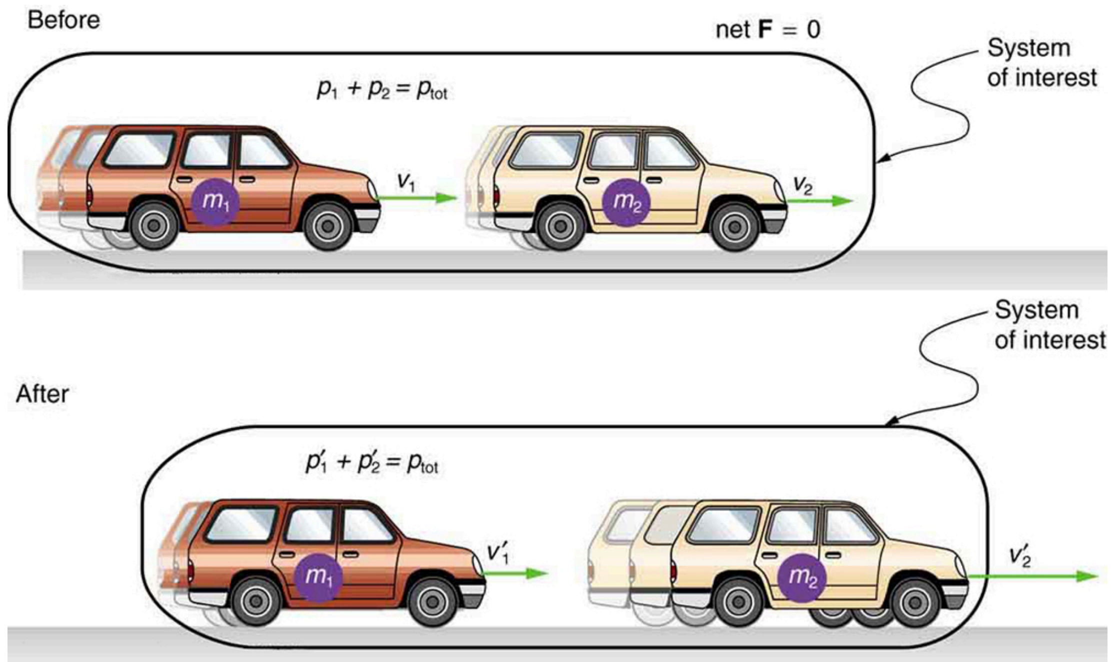


Figure 8.6 A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v'_1 and the second speeds up to a velocity of v'_2 . The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \tag{8.32}$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \tag{8.33}$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = \ -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \tag{8.34}$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0. \tag{8.35}$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant}, \tag{8.36}$$

$$p_1 + p_2 = p'_1 + p'_2, \tag{8.37}$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\mathbf{p}_{\text{tot}} = \text{constant}, \tag{8.38}$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}, \tag{8.39}$$

where $\mathbf{p}_{\text{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Conservation of Momentum Principle

$$\begin{aligned}\mathbf{p}_{\text{tot}} &= \text{constant} \\ \mathbf{p}_{\text{tot}} &= \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)}\end{aligned} \tag{8.40}$$

Isolated System

An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Making Connections: Cart Collisions

Consider two air carts with equal mass *(m)* on a linear track. The first cart moves with a speed *v* towards the second cart, which is initially at rest. We will take the initial direction of motion of the first cart as the positive direction.

The momentum of the system will be conserved in the collision. If the collision is elastic, then the first cart will stop after the collision. Conservation of momentum therefore tells us that the second cart will have a final velocity *v* after the collision in the same direction as the initial velocity of the first cart.

The kinetic energy of the system will be conserved since the masses are equal and the final velocity of cart 2 is equal to the initial velocity of cart 1. What would a graph of total momentum vs. time look like in this case? What would a graph of total kinetic energy vs. time look like in this case?

Consider the center of mass of this system as the frame of reference. As cart 1 approaches cart 2, the center of mass remains exactly halfway between the two carts. The center of mass moves toward the stationary cart 2 at a speed $\frac{v}{2}$. After the collision, the center of mass continues moving in the same direction, away from (now stationary) cart 1 at a speed $\frac{v}{2}$.

How would a graph of center-of-mass velocity vs. time compare to a graph of momentum vs. time?

Suppose instead that the two carts move with equal speeds *v* in opposite directions towards the center of mass. Again, they have an elastic collision, so after the collision, they exchange velocities (each cart moving in the opposite direction of its initial motion with the same speed). As the two carts approach, the center of mass is exactly between the two carts, at the point where they will collide. In this case, how would a graph of center-of-mass velocity vs. time compare to a graph of the momentum of the system vs. time?

Let us return to the example where the first cart is moving with a speed *v* toward the second cart, initially at rest. Suppose the second cart has some putty on one end so that, when the collision occurs, the two carts stick together in an inelastic collision. In this case, conservation of momentum tells us that the final velocity of the two-cart system will be half the initial velocity of the first cart, in the same direction as the first cart's initial motion. Kinetic energy will not be conserved in this case, however. Compared to the moving cart before the collision, the overall moving mass after the collision is doubled, but the velocity is halved.

The initial kinetic energy of the system is:

$$k_i = \frac{1}{2}mv^2(\text{1st cart}) + 0(\text{2nd cart}) = \frac{1}{2}mv^2 \tag{8.41}$$

The final kinetic energy of the two carts (2*m*) moving together (at speed *v*/2) is:

$$k_f = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2 \tag{8.42}$$

What would a graph of total momentum vs. time look like in this case? What would a graph of total kinetic energy vs. time look like in this case?

Consider the center of mass of this system. As cart 1 approaches cart 2, the center of mass remains exactly halfway between the two carts. The center of mass moves toward the stationary cart 2 at a speed $\frac{v}{2}$. After the collision, the two carts move together at a speed $\frac{v}{2}$. How would a graph of center-of-mass velocity vs. time compare to a graph of momentum vs. time?

Suppose instead that the two carts move with equal speeds *v* in opposite directions towards the center of mass. They have putty on the end of each cart so that they stick together after the collision. As the two carts approach, the center of mass is exactly between the two carts, at the point where they will collide. In this case, how would a graph of center-of-mass velocity vs. time compare to a graph of the momentum of the system vs. time?

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$. For an isolated system, $(\mathbf{F}_{\text{net}} = 0)$; thus, $\Delta \mathbf{p}_{\text{tot}} = 0$, and $\mathbf{p}_{\text{tot}}$ is constant.

We have noted that the three length dimensions in nature—x, y, and z—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See **Figure 8.7**.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

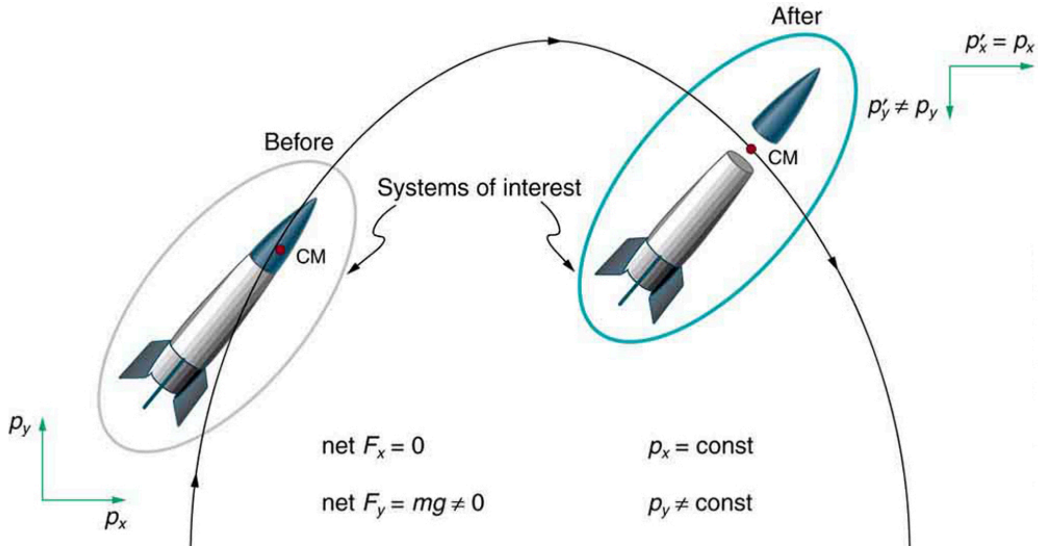


Figure 8.7 The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x-\text{net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y-\text{net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Applying Science Practices: Verifying the Conservation of Linear Momentum

Design an experiment to verify the conservation of linear momentum in a one-dimensional collision, both elastic and inelastic. For simplicity, try to ensure that friction is minimized so that it has a negligible effect on your experiment. As you consider your experiment, consider the following questions:

- Predict how the final momentum of the system will compare to the initial momentum of the system that you will measure. Justify your prediction.
- How will you measure the momentum of each object?
- Should you have two objects in motion or one object bouncing off a rigid surface?
- Should you verify the relationship mathematically or graphically?
- How will you estimate the uncertainty of your measurements? How will you express this uncertainty in your data?

When you have completed each experiment, compare the outcome to your prediction about the initial and final momentum of the system and evaluate your results.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. **Figure 8.8** below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

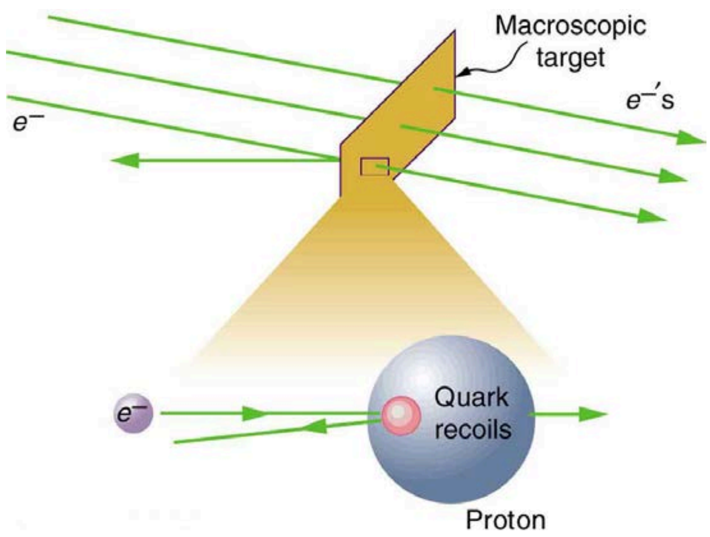


Figure 8.8 A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

8.4 Elastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one-dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.B.1.1** The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.). **(S.P. 1.4, 2.2)**
- **4.B.1.2** The student is able to analyze data to find the change in linear momentum for a constant-mass system using the product of the mass and the change in velocity of the center of mass. **(S.P. 5.1)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.1** The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. **(S.P. 6.4, 7.2)**
- **5.D.1.2** The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations. **(S.P. 2.2, 3.2, 5.1, 5.3)**
- **5.D.1.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.1.6** The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. **(S.P. 6.4)**
- **5.D.1.7** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.3.2** The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system. **(S.P. 6.4)**

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. **Figure 8.9** illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

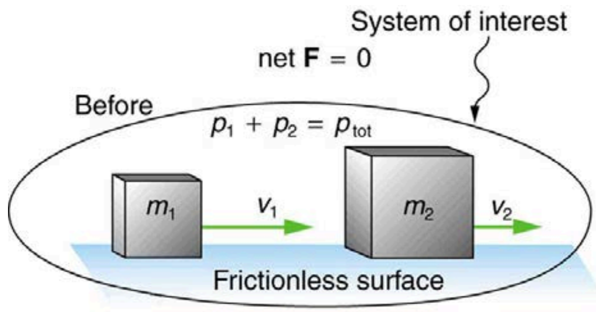
Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.



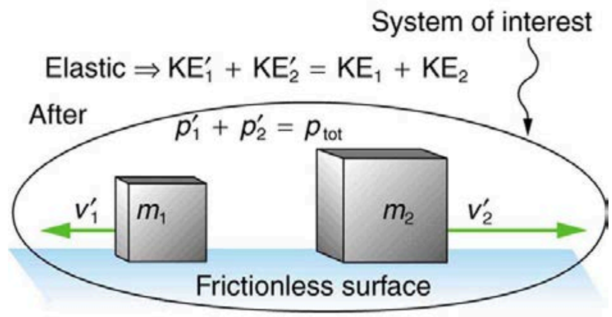


Figure 8.9 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \tag{8.43}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \tag{8.44}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2}m_1 {v_1}^2 + \frac{1}{2}m_2 {v_2}^2 = \frac{1}{2}m_1 {v'_1}^2 + \frac{1}{2}m_2 {v'_2}^2 \quad \text{(two-object elastic collision)} \tag{8.45}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Making Connections: Collisions

Suppose data are collected on a collision between two masses sliding across a frictionless surface. Mass A (1.0 kg) moves with a velocity of +12 m/s, and mass B (2.0 kg) moves with a velocity of −12 m/s. The two masses collide and stick together after the collision. The table below shows the measured velocities of each mass at times before and after the collision:

Table 8.1

Time (s)	Velocity A (m/s)	Velocity B (m/s)
0	+12	−12
1.0 s	+12	−12
2.0 s	−4.0	−4.0
3.0 s	−4.0	−4.0

The total mass of the system is 3.0 kg. The velocity of the center of mass of this system can be determined from the conservation of momentum. Consider the system before the collision:

$$(m_A + m_B)v_{cm} = m_A v_A + m_B v_B \tag{8.46}$$

$$(3.0)v_{cm} = (1)(12) + (2)(-12) \tag{8.47}$$

$$v_{cm} = -4.0 \text{ m/s} \tag{8.48}$$

After the collision, the center-of-mass velocity is the same:

$$(m_A + m_B)v_{cm} = (m_A + m_B)v_{final} \tag{8.49}$$

$$(3.0)v_{cm} = (3)(-4.0) \tag{8.50}$$

$$v_{cm} = -4.0\text{ m/s} \tag{8.51}$$

The total momentum of the system before the collision is:

$$m_A v_A + m_B v_B = (1)(12) + (2)(-12) = -12\text{ kg} \bullet \text{m/s} \tag{8.52}$$

The total momentum of the system after the collision is:

$$(m_A + m_B)v_{final} = (3)(-4) = -12\text{ kg} \bullet \text{m/s} \tag{8.53}$$

Thus, the change in momentum of the system is zero when measured this way. We get a similar result when we calculate the momentum using the center-of-mass velocity. Since the center-of-mass velocity is the same both before and after the collision, we calculate the same momentum for the system using this method both before and after the collision.

Example 8.4 Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500\text{ kg},\ m_2 = 3.50\text{ kg},\ v_1 = 4.00\text{ m/s, and }\ v_2 = 0. \tag{8.54}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in **Figure 8.9** where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{8.55}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{8.56}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2}m_1 {v_1}^2 = \frac{1}{2}m_1 {v'_1}^2 + \frac{1}{2}m_2 {v'_2}^2. \tag{8.57}$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{8.58}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00\text{ m/s} \tag{8.59}$$

and

$$v'_1 = -3.00\text{ m/s}. \tag{8.60}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00\text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}[4.00 - (-3.00)] \text{ m/s} \tag{8.61}$$

or

$$v'_2 = 1.00 \text{ m/s}. \tag{8.62}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PhET Explorations: Collision Lab

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

Figure 8.10 Collision Lab (http://cnx.org/content/m55171/1.7/collision-lab_en.jar)

8.5 Inelastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Define inelastic collision.
- Explain perfectly inelastic collisions.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

The information presented in this section supports the following AP® learning objectives and science practices:

- **4.B.1.1** The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.). **(S.P. 1.4, 2.2)**
- **5.A.2.1** The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **(S.P. 6.4, 7.2)**
- **5.D.1.3** The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. **(S.P. 2.1, 2.2)**
- **5.D.1.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **(S.P. 2.1, 2.2)**
- **5.D.2.1** The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **(S.P. 6.4, 7.2)**
- **5.D.2.2** The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **(S.P.4.1, 4.2, 5.1)**
- **5.D.2.3** The student is able to apply the conservation of linear momentum to a closed system of objects involved in an

inelastic collision to predict the change in kinetic energy. **(S.P. 6.4, 7.2)**

- **5.D.2.4** The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force. **(S.P. 4.1, 4.2, 4.4, 5.1, 5.3)**
- **5.D.2.5** The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. **(S.P. 2.1 2.2)**
- **5.D.2.6** The student is able to apply the conservation of linear momentum to an isolated system of objects involved in an inelastic collision to predict the change in kinetic energy. **(S.P. 6.4, 7.2)**

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 8.11 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called “perfectly inelastic.”

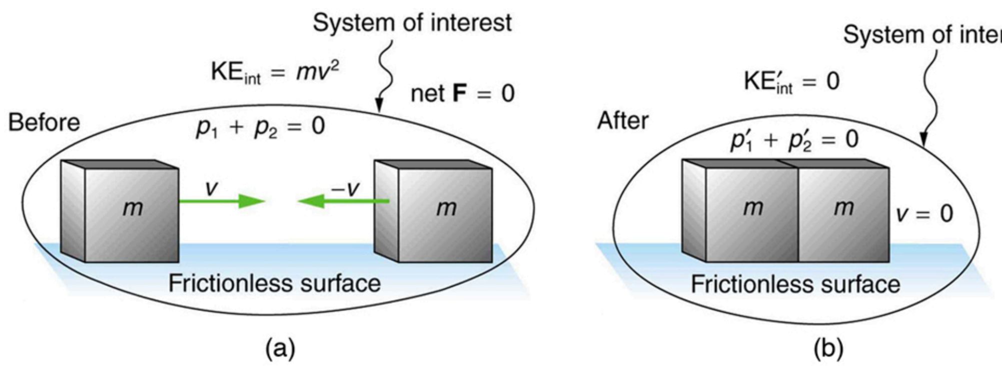


Figure 8.11 An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Example 8.5 Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See **Figure 8.12**)

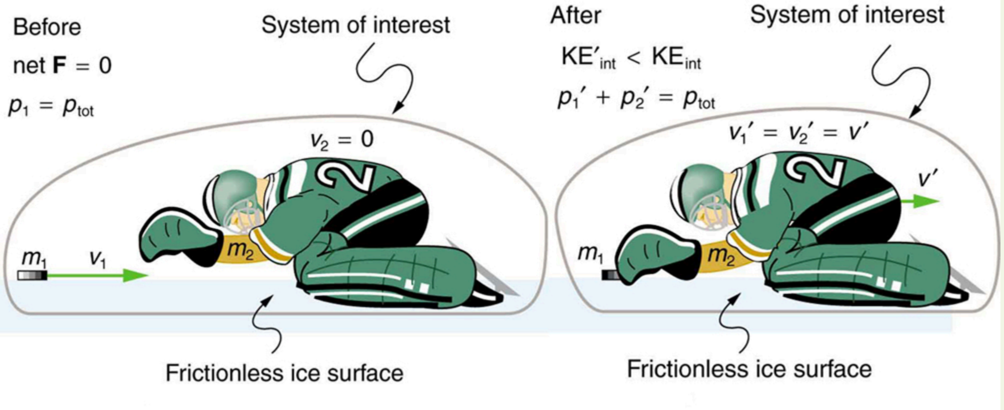


Figure 8.12 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \tag{8.63}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.64}$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2)v'. \tag{8.65}$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \tag{8.66}$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}. \tag{8.67}$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned} \tag{8.68}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}(m + M)v^2 = \frac{1}{2}(70.15 \text{ kg})\left(7.48 \times 10^{-2} \text{ m/s}\right)^2 \\ &= 0.196 \text{ J}. \end{aligned} \tag{8.69}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \mathrm{KE'}_{\text{int}} - \mathrm{KE}_{\text{int}} &= 0.196\text{ J} - 91.9\text{ J} \\ &= -91.7\text{ J} \end{aligned} \quad (8.70)$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. **Figure 8.13** shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. **Example 8.6** deals with data from such a collision.

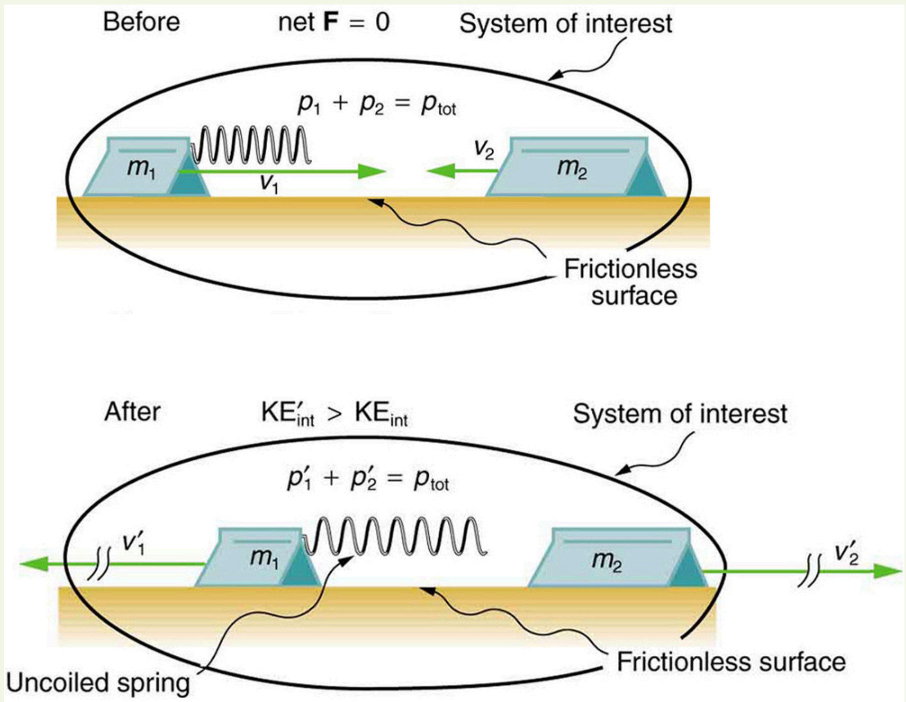


Figure 8.13 An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in **Example 8.6**, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and

for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 8.6 Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in **Figure 8.13**, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in **Figure 8.13**) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.71}$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s})}{0.500 \text{ kg}} - \frac{(0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}} \\ &= 3.70 \text{ m/s}. \end{aligned} \tag{8.72}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \\ &= \frac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2 \\ &= 0.763 \text{ J}. \end{aligned} \tag{8.73}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2 \\ &= \frac{1}{2}(0.350 \text{ kg})(\text{-}4.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(3.70 \text{ m/s})^2 \\ &= 6.22 \text{ J}. \end{aligned} \tag{8.74}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22 \text{ J} - 0.763 \text{ J} \\ &= 5.46 \text{ J}. \end{aligned} \tag{8.75}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

8.6 Collisions of Point Masses in Two Dimensions

Learning Objectives

By the end of this section, you will be able to:

- Discuss two-dimensional collisions as an extension of one-dimensional analysis.

- Define point masses.
- Derive an expression for conservation of momentum along the *x*-axis and *y*-axis.
- Describe elastic collisions of two objects with equal mass.
- Determine the magnitude and direction of the final velocity given initial velocity and scattering angle.

The information presented in this section supports the following AP® learning objectives and science practices:

- **5.D.1.2** The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations.
- **5.D.3.3** The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system.

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. (See **Figure 8.14**.) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in **Figure 8.14**. Because momentum is conserved, the components of momentum along the x - and y -axes $(p_x \text{ and } p_y)$ will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

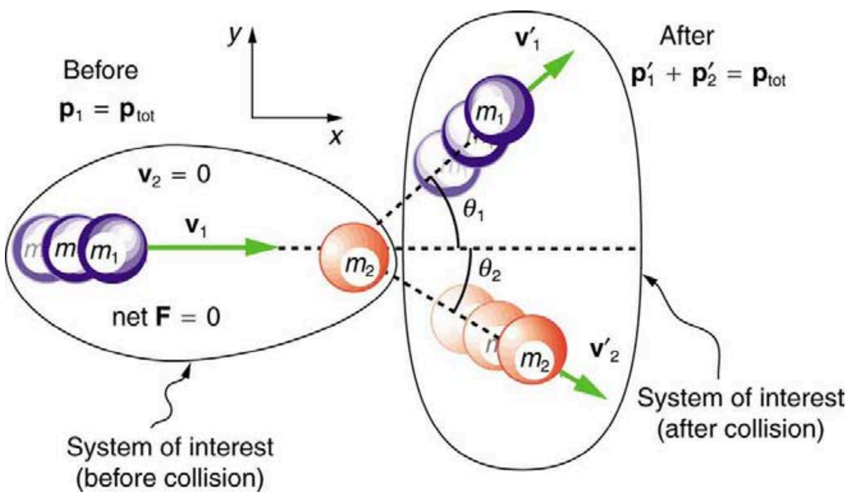


Figure 8.14 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x -axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

Along the x -axis, the equation for conservation of momentum is

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}. \tag{8.76}$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{8.77}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{8.78}$$

The components of the velocities along the x-axis have the form $v \cos \theta$. Because particle 1 initially moves along the x-axis, we find $v_{1x} = v_1$.

Conservation of momentum along the x-axis gives the following equation:

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2, \tag{8.79}$$

where θ_1 and θ_2 are as shown in **Figure 8.14**.

Conservation of Momentum along the x-axis

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2 \tag{8.80}$$

Along the y-axis, the equation for conservation of momentum is

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y} \tag{8.81}$$

or

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}. \tag{8.82}$$

But v_{1y} is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 v'_{1y} + m_2 v'_{2y}. \tag{8.83}$$

The components of the velocities along the y-axis have the form $v \sin \theta$.

Thus, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2. \tag{8.84}$$

Conservation of Momentum along the y-axis

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2 \tag{8.85}$$

The equations of conservation of momentum along the x-axis and y-axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

Making Connections: Real World Connections

We have seen, in one-dimensional collisions when momentum is conserved, that the center-of-mass velocity of the system remains unchanged as a result of the collision. If you calculate the momentum and center-of-mass velocity before the collision, you will get the same answer as if you calculate both quantities after the collision. This logic also works for two-dimensional collisions.

For example, consider two cars of equal mass. Car A is driving east (+*x*-direction) with a speed of 40 m/s. Car B is driving north (+*y*-direction) with a speed of 80 m/s. What is the velocity of the center-of-mass of this system before and after an inelastic collision, in which the cars move together as one mass after the collision?

Since both cars have equal mass, the center-of-mass velocity components are just the average of the components of the individual velocities before the collision. The *x*-component of the center of mass velocity is 20 m/s, and the y-component is 40 m/s.

Using momentum conservation for the collision in both the *x*-component and *y*-component yields similar answers:

$$m(40) + m(0) = (2m)v_{\text{fina}}(x) \tag{8.86}$$

$$v_{\text{fina}}(x) = 20 \text{ m/s} \tag{8.87}$$

$$m(0) + m(80) = (2m)v_{\text{fina}}(y) \tag{8.88}$$

$$v_{\text{fina}}(y) = 40 \text{ m/s} \tag{8.89}$$

Since the two masses move together after the collision, the velocity of this combined object is equal to the center-of-mass velocity. Thus, the center-of-mass velocity before and after the collision is identical, even in two-dimensional collisions, when momentum is conserved.

Example 8.7 Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0° with its incoming direction.

The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity $(v'_2$ and $\theta_2)$ of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in **Figure 8.15** is one in which m_2 is originally at rest and the initial velocity is parallel to the x-axis, so that conservation of momentum along the x- and y-axes is applicable.

Everything is known in these equations except v'_2 and θ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x- and y-directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ for $v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for $v'_2 \sin\theta_2$ and taking the ratio yields an equation (in which θ_2 is the only unknown quantity. Applying the identity $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$, we obtain:

$$\tan\theta_2 = \frac{v'_1 \sin\theta_1}{v'_1 \cos\theta_1 - v_1}. \quad (8.90)$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50 \text{ m/s})(0.7071)}{(1.50 \text{ m/s})(0.7071) - 2.00 \text{ m/s}} = -1.129. \quad (8.91)$$

Thus,

$$\theta_2 = \tan^{-1}(-1.129) = 311.5^\circ \approx 312^\circ. \quad (8.92)$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in **Figure 8.15**, as expected (this angle is in the fourth quadrant). Either equation for the x- or y-axis can now be used to solve for v'_2, but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\frac{m_1}{m_2} v'_1 \frac{\sin\theta_1}{\sin\theta_2} \quad (8.93)$$

Entering known values into this equation gives

$$v'_2 = -\left(\frac{0.250 \text{ kg}}{0.400 \text{ kg}}\right)(1.50 \text{ m/s})\left(\frac{0.7071}{-0.7485}\right). \quad (8.94)$$

Thus,

$$v'_2 = 0.886 \text{ m/s}. \quad (8.95)$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

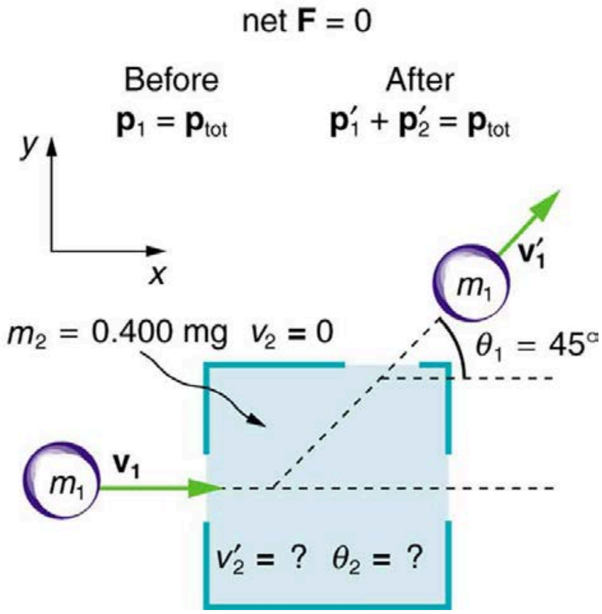


Figure 8.15 A collision taking place in a dark room is explored in **Example 8.7**. The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to **Figure 8.14** for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 (m_2) is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2. \tag{8.96}$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x- and y-directions can show that

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1v'_2\cos(\theta_1 - \theta_2). \tag{8.97}$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv'_1v'_2\cos(\theta_1 - \theta_2) = 0. \tag{8.98}$$

There are three ways that this term can be zero. They are

- $v'_1 = 0$: head-on collision; incoming ball stops
- $v'_2 = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation $(\theta_1 - \theta_2)$ is 90° after the collision

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to 90° after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

Connections to Nuclear and Particle Physics

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in **Medical Applications of Nuclear Physics** and **Particle Physics**. Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

8.7 Introduction to Rocket Propulsion

Learning Objectives

By the end of this section, you will be able to:

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket.
- Discuss the factors that affect the rocket's acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

Making Connections: Take-Home Experiment—Propulsion of a Balloon

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

Figure 8.16 shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv . In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_{e} relative to the rocket. The remainder of the mass $(m - \Delta m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt , producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over Δt , and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_{\text{e}}}{m}\frac{\Delta m}{\Delta t} - g \tag{8.99}$$

"The rocket" is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

Acceleration of a Rocket

Acceleration of a rocket is

$$a = \frac{v_{\text{e}}}{m}\frac{\Delta m}{\Delta t} - g, \tag{8.100}$$

where a is the acceleration of the rocket, v_{e} is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

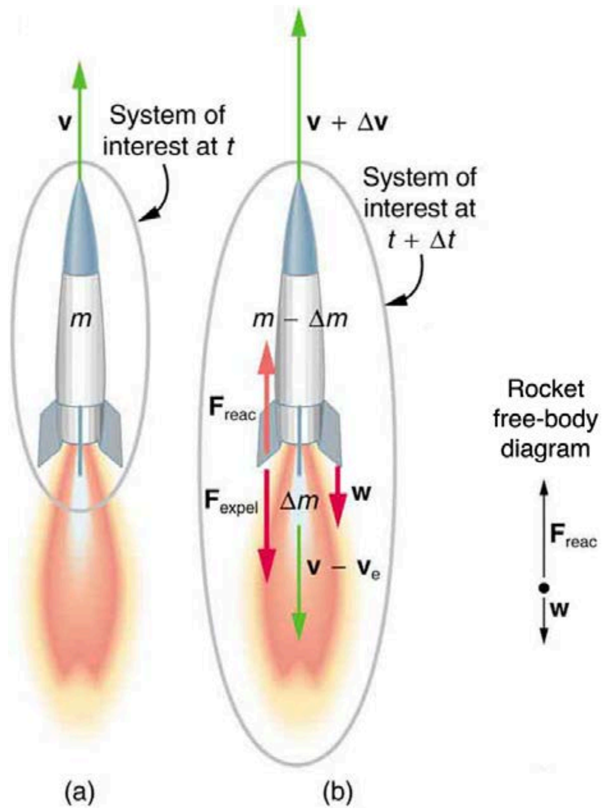


Figure 8.16 (a) This rocket has a mass m and an upward velocity v. The net external force on the system is $-mg$, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket **.** First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about $2.5\times10^3\ \text{m/s}$ for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\Delta m / \Delta t$ in the equation. The quantity $(\Delta m / \Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

Example 8.8 Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was $2.80\times10^6\ \text{kg}$, its fuel-burn rate was $1.40\times10^4\ \text{kg/s}$, and the exhaust velocity was $2.40\times10^3\ \text{m/s}$. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g = \frac{2.40\times10^3\ \text{m/s}}{2.80\times10^6\ \text{kg}}\left(1.40\times10^4\ \text{kg/s}\right) - 9.80\ \text{m/s}^2 = 2.20\ \text{m/s}^2. \quad (8.101)$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was $3.36\times10^7\ \text{N}$.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln\frac{m_0}{m_r}, \quad (8.102)$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about $11.2\times10^3\ \text{m/s}$, and assuming an exhaust velocity $v_e = 2.5\times10^3\ \text{m/s}$.

$$\ln\frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2\times10^3\ \text{m/s}}{2.5\times10^3\ \text{m/s}} = 4.48 \quad (8.103)$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88. \quad (8.104)$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}. \quad (8.105)$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See **Figure 8.17**) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

Figure 8.17 The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

PhET Explorations: Lunar Lander

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

PhET Interactive Simulation

Figure 8.18 Lunar Lander (http://cnx.org/content/m55174/1.2/lunar-lander_en.jar)

Glossary

change in momentum: the difference between the final and initial momentum; the mass times the change in velocity

conservation of momentum principle: when the net external force is zero, the total momentum of the system is conserved or constant

elastic collision: a collision that also conserves internal kinetic energy

impulse: the average net external force times the time it acts; equal to the change in momentum

inelastic collision: a collision in which internal kinetic energy is not conserved

internal kinetic energy: the sum of the kinetic energies of the objects in a system

isolated system: a system in which the net external force is zero

linear momentum: the product of mass and velocity

perfectly inelastic collision: a collision in which the colliding objects stick together

point masses: structureless particles with no rotation or spin

quark: fundamental constituent of matter and an elementary particle

second law of motion: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Section Summary

8.1 Linear Momentum and Force

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be

$$\mathbf{p} = m\mathbf{v},$$

where m is the mass of the system and $\mathbf{v}$ is its velocity.
- The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t},$$

$\mathbf{F}_{\text{net}}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change time.

8.2 Impulse

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t.$$

- Forces are usually not constant over a period of time.

8.3 Conservation of Momentum

- The conservation of momentum principle is written

$$\mathbf{p}_{\text{tot}} = \text{constant}$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)},$$

$\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.
- An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

8.4 Elastic Collisions in One Dimension

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

8.5 Inelastic Collisions in One Dimension

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

8.6 Collisions of Point Masses in Two Dimensions

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1 v'_{1y} + m_2 v'_{2y}$.
- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^{\ 2} = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2).$$

- Point masses are structureless particles that cannot spin.

8.7 Introduction to Rocket Propulsion

- Newton's third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$.
- A rocket's acceleration depends on three main factors. They are
  1. The greater the exhaust velocity of the gases, the greater the acceleration.
  2. The faster the rocket burns its fuel, the greater its acceleration.
  3. The smaller the rocket's mass, the greater the acceleration.

Conceptual Questions

8.1 Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3. Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

4. How can a small force impart the same momentum to an object as a large force?

8.2 Impulse

5. Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

6. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

7. Professional Application

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

8.3 Conservation of Momentum

8. Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

9. Under what circumstances is momentum conserved?

10. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

11. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

12. Professional Application

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

13. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

14. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

8.4 Elastic Collisions in One Dimension

15. What is an elastic collision?

8.5 Inelastic Collisions in One Dimension

16. What is an inelastic collision? What is a perfectly inelastic collision?

17. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

18. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

8.6 Collisions of Point Masses in Two Dimensions

19. Figure 8.19 shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b , the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.

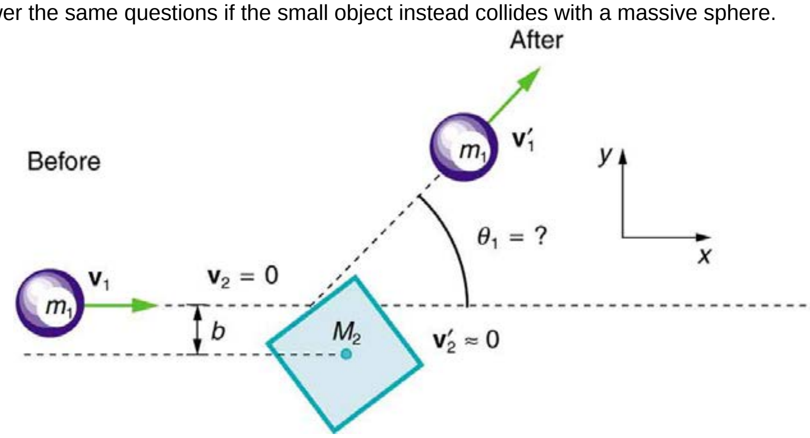


Figure 8.19 A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1 . The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b .

8.7 Introduction to Rocket Propulsion

20. Professional Application

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

21. Professional Application

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

22. Professional Application

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

8.1 Linear Momentum and Force

1. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

2. (a) What is the mass of a large ship that has a momentum of $1.60\times10^9 \text{ kg}\cdot\text{m/s}$, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

3. (a) At what speed would a $2.00\times10^4\text{-kg}$ airplane have to fly to have a momentum of $1.60\times10^9 \text{ kg}\cdot\text{m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

4. (a) What is the momentum of a garbage truck that is $1.20\times10^4 \text{ kg}$ and is moving at 10.0 m/s? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

5. A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.

6. The mass of Earth is $5.972\times10^{24} \text{ kg}$ and its orbital radius is an average of $1.496\times10^{11} \text{ m}$. Calculate its linear momentum.

8.2 Impulse

7. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

8. Professional Application

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

9. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

10. Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

11. Professional Application

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

12. Professional Application

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00\times10^3 \text{ m/s}$, given the collision lasts $6.00\times10^{-8} \text{ s}$.

13. Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

14. Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

15. A cruise ship with a mass of $1.00\times10^7 \text{ kg}$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

16. Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76\times10^4 \text{ N}$ for $5.50\times10^{-2} \text{ s}$.

17. Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

18. A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

19. Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

20. A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$ -direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$ -direction with the same speed. What is the impulse delivered by the wall?

21. When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

22. A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

8.3 Conservation of Momentum

23. Professional Application

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s . (The minus indicates direction of motion.) What is their final velocity?

24. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

25. Professional Application

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

26. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

27. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

8.4 Elastic Collisions in One Dimension

28. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

29. Professional Application

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^3\text{ kg}$, and the second a mass of $7.50\times10^3\text{ kg}$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

30. A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

8.5 Inelastic Collisions in One Dimension

31. A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

32. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

33. Professional Application

Using mass and speed data from **Example 8.1** and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

34. A battleship that is $6.00\times10^7\text{ kg}$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

35. Professional Application

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^3\text{ kg}$, and the second a mass of $7.50\times10^3\text{ kg}$.

(a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

36. Professional Application

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

37. Professional Application

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

38. A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

39. Professional Application

One of the waste products of a nuclear reactor is plutonium-239 $\left({}^{239}\text{Pu}\right)$. This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus $\left({}^{4}\text{He} + {}^{235}\text{U}\right)$, the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is $8.40\times10^{-13}\ \text{J}$ and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68\times10^{-27}\ \text{kg}$, while that of the uranium is $3.92\times10^{-25}\ \text{kg}$ (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

40. Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of $5.00\times10^{12}\ \text{kg}$ (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36\times10^{22}\ \text{kg}$) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

41. Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

42. What is the speed of a garbage truck that is $1.20\times10^{4}\ \text{kg}$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

43. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

44. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

8.6 Collisions of Point Masses in Two Dimensions

45. Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0° ,what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^\circ$ for elastic collisions of objects that have identical masses.) (b) Confirm that the collision is elastic.

46. Confirm that the results of the example **Example 8.7** do conserve momentum in both the x - and y -directions.

47. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

48. Professional Application

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0° to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic? (c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

49. Professional Application

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei $\left(^{4}\text{He}\right)$ from gold-197 nuclei $\left(^{197}\text{Au}\right)$. The energy of the incoming helium nucleus was $8.00\times10^{-13}\text{ J}$, and the masses of the helium and gold nuclei were $6.68\times10^{-27}\text{ kg}$ and $3.29\times10^{-25}\text{ kg}$, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?

50. Professional Application

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at 8.00 m/s due south. The second car has a mass of 850 kg and is approaching at 17.0 m/s due west. (a) Calculate the final velocity (magnitude and direction) of the cars. (b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

51. Starting with equations $m_1v_1 = m_1v'_1\cos\theta_1 + m_2v'_2\cos\theta_2$ and $0 = m_1v'_1\sin\theta_1 + m_2v'_2\sin\theta_2$ for conservation of momentum in the x- and y-directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2}mv_1^{\ 2} = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1v'_2\cos\left(\theta_1 - \theta_2\right)$$

as discussed in the text.

52. Integrated Concepts

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

8.7 Introduction to Rocket Propulsion

53. Professional Application

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50\times10^{3}\text{ m/s?}$

54. Professional Application

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only 1.6 m/s^2, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20\times10^{3}\text{ m/s?}$

55. Professional Application

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00\times10^{3}\text{ m/s}$. You may assume the gravitational force is negligible at the probe's location.

56. Professional Application

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00\times10^{6}\text{ m/s}$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50\times10^{-6}\text{ kg/s}$ at the given velocity, assuming the acceleration due to gravity is negligible.

57. Derive the equation for the vertical acceleration of a rocket.

58. Professional Application

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is $2.40\times10^{3}\text{ m/s}$. Assume that the acceleration of gravity is the same as on Earth's surface $\left(9.80\text{ m/s}^2\right)$. (b) Why might it be necessary to limit the acceleration of a rocket?

59. Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

60. How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of 8.00 km/s, given that it expels gases at an exhaust velocity of $2.20\times10^{3}\text{ m/s?}$

61. Professional Application

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?

62. Unreasonable Results

Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

63. Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

64. Construct Your Own Problem

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

Test Prep for AP® Courses

8.1 Linear Momentum and Force

1. A boy standing on a frictionless ice rink is initially at rest. He throws a snowball in the +*x*-direction, and it travels on a ballistic trajectory, hitting the ground some distance away. Which of the following is true about the boy while he is in the act of throwing the snowball?

a. He feels an upward force to compensate for the downward trajectory of the snowball.
b. He feels a backward force exerted by the snowball he is throwing.
c. He feels no net force.
d. He feels a forward force, the same force that propels the snowball.

2. A 150-g baseball is initially moving 80 mi/h in the –*x*-direction. After colliding with a baseball bat for 20 ms, the baseball moves 80 mi/h in the +*x*-direction. What is the magnitude and direction of the average force exerted by the bat on the baseball?

8.2 Impulse

3. A 1.0-kg ball of putty is released from rest and falls vertically 1.5 m until it strikes a hard floor, where it comes to rest in a 0.045-s time interval. What is the magnitude and direction of the average force exerted on the ball by the floor during the collision?

a. 33 N, up
b. 120 N, up
c. 120 N, down
d. 240 N, down

4. A 75-g ball is dropped from rest from a height of 2.2 m. It bounces off the floor and rebounds to a maximum height of 1.7 m. If the ball is in contact with the floor for 0.024 s, what is the magnitude and direction of the average force exerted on the ball by the floor during the collision?

5. A 2.4-kg ceramic bowl falls to the floor. During the 0.018-s impact, the bowl experiences an average force of 750 N from the floor. The bowl is at rest after the impact. From what initial height did the bowl fall?

a. 1.6 m
b. 2.8 m
c. 3.2 m
d. 5.6 m

6. Whether or not an object (such as a plate, glass, or bone) breaks upon impact depends on the average force exerted on that object by the surface. When a 1.2-kg glass figure hits the floor, it will break if it experiences an average force of 330 N. When it hits a tile floor, the glass comes to a stop in 0.015 s. From what minimum height must the glass fall to experience sufficient force to break? How would your answer change if the figure were falling to a padded or carpeted surface? Explain.

7. A 2.5-kg block slides across a frictionless table toward a horizontal spring.As the block bounces off the spring, a probe measures the velocity of the block (initially negative, moving away from the probe) over time as follows:

Table 8.2

Velocity (m/s)	Time (s)
−12.0	0
−10.0	0.10
−6.0	0.20
0	0.30
6.0	0.40
10.0	0.50
12.0	0.60

What is the average force exerted on the block by the spring over the entire 0.60-s time interval of the collision?

a. 50 N
b. 60 N
c. 100 N
d. 120 N

8. During an automobile crash test, the average force exerted by a solid wall on a 2500-kg car that hits the wall is measured to be 740,000 N over a 0.22-s time interval. What was the initial speed of the car prior to the collision, assuming the car is at rest at the end of the time interval?

9. A test car is driving toward a solid crash-test barrier with a speed of 45 mi/h. Two seconds prior to impact, the car begins to brake, but it is still moving when it hits the wall. After the collision with the wall, the car crumples somewhat and comes to a complete stop. In order to estimate the average force exerted by the wall on the car, what information would you need to collect?

a. The (negative) acceleration of the car before it hits the wall and the distance the car travels while braking.
b. The (negative) acceleration of the car before it hits the wall and the velocity of the car just before impact.
c. The velocity of the car just before impact and the duration of the collision with the wall.
d. The duration of the collision with the wall and the distance the car travels while braking.

10. Design an experiment to verify the relationship between the average force exerted on an object and the change in momentum of that object. As part of your explanation, list the equipment you would use and describe your experimental setup. What would you measure and how? How exactly would you verify the relationship? Explain.

11. A 22-g puck hits the wall of an air hockey table perpendicular to the wall with an initial speed of 14 m/s.The puck is in contact with the wall for 0.0055 s, and it rebounds from the wall with a speed of 14 m/s in the opposite direction.What is the magnitude of the average force exerted by the wall on the puck?

a. 0.308 N
b. 0.616 N
c. 56 N
d. 112 N

12. A 22-g puck hits the wall of an air hockey table perpendicular to the wall with an initial speed of 7 m/s. The puck is in contact with the wall for 0.011 s, and the wall exerts an average force of 28 N on the puck during that time. Calculate the magnitude and direction of the change in momentum of the puck.

13.

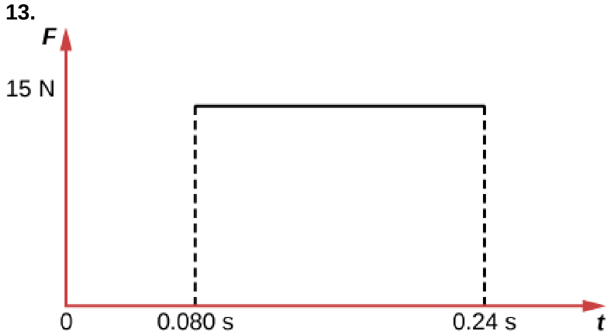


Figure 8.20 This is a graph showing the force exerted by a rigid wall versus time. The graph in **Figure 8.20** represents the force exerted on a particle during a collision. What is the magnitude of the change in momentum of the particle as a result of the collision?

a. 1.2 kg • m/s
b. 2.4 kg • m/s
c. 3.6 kg • m/s
d. 4.8 kg • m/s

14.

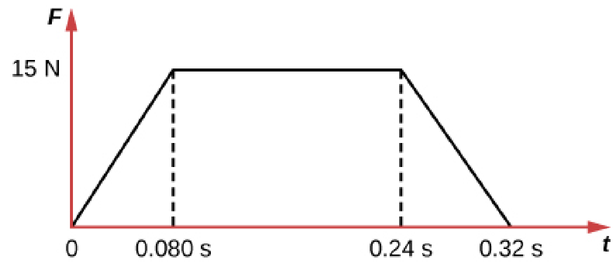


Figure 8.21 This is a graph showing the force exerted by a rigid wall versus time. The graph in **Figure 8.21** represents the force exerted on a particle during a collision. What is the magnitude of the change in momentum of the particle as a result of the collision?

8.3 Conservation of Momentum

15. Which of the following is an example of an open system?

a. Two air cars colliding on a track elastically.
b. Two air cars colliding on a track and sticking together.
c. A bullet being fired into a hanging wooden block and becoming embedded in the block, with the system then acting as a ballistic pendulum.
d. A bullet being fired into a hillside and becoming buried in the earth.

16. A 40-kg girl runs across a mat with a speed of 5.0 m/s and jumps onto a 120-kg hanging platform initially at rest, causing the girl and platform to swing back and forth like a pendulum together after her jump. What is the combined velocity of the girl and platform after the jump? What is the combined momentum of the girl and platform both before and after the collision?

A 50-kg boy runs across a mat with a speed of 6.0 m/s and collides with a soft barrier on the wall, rebounding off the wall and falling to the ground. The boy is at rest after the collision. What is the momentum of the boy before and after the collision? Is momentum conserved in this collision? Explain. Which of these is an example of an open system and which is an example of a closed system? Explain your answer.

17. A student sets up an experiment to measure the momentum of a system of two air cars, A and B, of equal mass, moving on a linear, frictionless track. Before the collision, car A has a certain speed, and car B is at rest. Which of the following will be true about the total momentum of the two cars?

a. It will be greater before the collision.
b. It will be equal before and after the collision.
c. It will be greater after the collision.
d. The answer depends on whether the collision is elastic or inelastic.

18. A group of students has two carts, *A* and *B*, with wheels that turn with negligible friction. The carts can travel along a straight horizontal track. Cart *A* has known mass *mA*. The students are asked to use a one-dimensional collision between the carts to determine the mass of cart *B*. Before the collision, cart *A* travels to the right and cart *B* is initially at rest. After the collision, the carts stick together.

a. Describe an experimental procedure to determine the velocities of the carts before and after a collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.
b. There will be sources of error in the measurements taken in the experiment, both before and after the collision. For your experimental procedure, will the uncertainty in the calculated value of the mass of cart *B* be affected more by the error in the measurements taken before the collision or by those taken after the collision, or will it be equally affected by both sets of measurements? Justify your answer.

A group of students took measurements for one collision. A graph of the students' data is shown below.

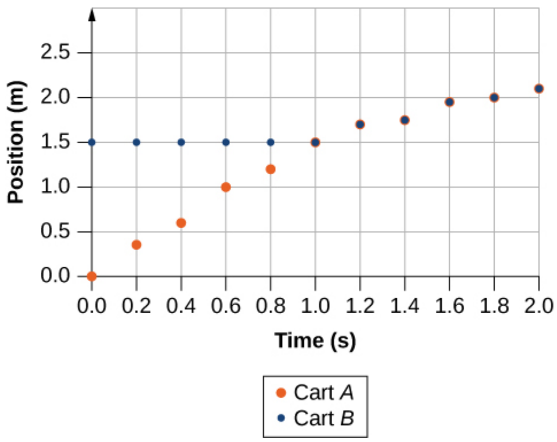


Figure 8.22 The image shows a graph with position in meters on the vertical axis and time in seconds on the horizontal axis.

c. Given m_A = 0.50 kg, use the graph to calculate the mass of cart *B*. Explicitly indicate the principles used in your calculations.
d. The students are now asked to Consider the kinetic energy changes in an inelastic collision, specifically whether the initial values of one of the physical quantities affect the fraction of mechanical energy dissipated in the collision. How could you modify the experiment to investigate this question? Be sure to explicitly describe the calculations you would make, specifying all equations you would use (but do not actually do any algebra or arithmetic).

19. Cart A is moving with an initial velocity +*v* (in the positive direction) toward cart B, initially at rest. Both carts have equal mass and are on a frictionless surface. Which of the following statements correctly characterizes the velocity of the center of mass of the system before and after the collision?

a. $\frac{+v}{2}$ before, $\frac{-v}{2}$ after
b. $\frac{+v}{2}$ before, 0 after
c. $\frac{+v}{2}$ before, $\frac{+v}{2}$ after
d. 0 before, 0 after

20. Cart A is moving with a velocity of +10 m/s toward cart B, which is moving with a velocity of +4 m/s. Both carts have equal mass and are moving on a frictionless surface. The two carts have an inelastic collision and stick together after the collision. Calculate the velocity of the center of mass of the system before and after the collision. If there were friction present in this problem, how would this external force affect the center-of-mass velocity both before and after the collision?

8.4 Elastic Collisions in One Dimension

21. Two cars (A and B) of mass 1.5 kg collide. Car A is initially moving at 12 m/s, and car B is initially moving in the same direction with a speed of 6 m/s. The two cars are moving along a straight line before and after the collision. What will be the change in momentum of this system after the collision?

a. −27 kg • m/s
b. zero
c. +27 kg • m/s
d. It depends on whether the collision is elastic or inelastic.

22. Two cars (A and B) of mass 1.5 kg collide. Car A is initially moving at 24 m/s, and car B is initially moving in the opposite direction with a speed of 12 m/s. The two cars are moving along a straight line before and after the collision. (a) If the two cars have an elastic collision, calculate the change in momentum of the two-car system. (b) If the two cars have a completely inelastic collision, calculate the change in momentum of the two-car system.

23. Puck A (200 g) slides across a frictionless surface to collide with puck B (800 g), initially at rest. The velocity of each puck is measured during the experiment as follows:

Table 8.3

Time	Velocity A	Velocity B
0	+8.0 m/s	0
1.0 s	+8.0 m/s	0
2.0 s	−2.0 m/s	+2.5 m/s
3.0 s	−2.0 m/s	+2.5 m/s

What is the change in momentum of the center of mass of the system as a result of the collision?

a. +1.6 kg•m/s
b. +0.8 kg•m/s
c. 0
d. −1.6 kg•m/s

24. For the table above, calculate the center-of-mass velocity of the system both before and after the collision, then calculate the center-of-mass momentum of the system both before and after the collision. From this, determine the change in the momentum of the system as a result of the collision.

25. Two cars (A and B) of equal mass have an elastic collision. Prior to the collision, car A is moving at 15 m/s in the +*x*-direction, and car B is moving at 10 m/s in the –*x*-direction. Assuming that both cars continue moving along the *x*-axis after the collision, what will be the velocity of car A after the collision?

a. same as the original 15 m/s speed, opposite direction
b. equal to car B's velocity prior to the collision
c. equal to the average of the two velocities, in its original direction
d. equal to the average of the two velocities, in the opposite direction

26. Two cars (A and B) of equal mass have an elastic collision. Prior to the collision, car A is moving at 20 m/s in the +*x*-direction, and car B is moving at 10 m/s in the –*x*-direction. Assuming that both cars continue moving along the *x*-axis after the collision, what will be the velocities of each car after the collision?

27. A rubber ball is dropped from rest at a fixed height. It bounces off a hard floor and rebounds upward, but it only reaches 90% of its original fixed height. What is the best way to explain the loss of kinetic energy of the ball during the collision?

a. Energy was required to deform the ball's shape during the collision with the floor.
b. Energy was lost due to work done by the ball pushing on the floor during the collision.
c. Energy was lost due to friction between the ball and the floor.
d. Energy was lost due to the work done by gravity during the motion.

28. A tennis ball strikes a wall with an initial speed of 15 m/s. The ball bounces off the wall but rebounds with slightly less speed (14 m/s) after the collision. Explain (a) what else changed its momentum in response to the ball's change in momentum so that overall momentum is conserved, and (b) how some of the ball's kinetic energy was lost.

29. Two objects, A and B, have equal mass. Prior to the collision, mass A is moving 10 m/s in the +*x*-direction, and mass B is moving 4 m/s in the +*x*-direction. Which of the following results represents an inelastic collision between A and B?

a. After the collision, mass A is at rest, and mass B moves 14 m/s in the +*x*-direction.
b. After the collision, mass A moves 4 m/s in the –*x*-direction, and mass B moves 18 m/s in the +*x*-direction.
c. After the collision, the two masses stick together and move 7 m/s in the +*x*-direction.
d. After the collision, mass A moves 4 m/s in the +*x*-direction, and mass B moves 10 m/s in the +*x*-direction.

30. Mass A is three times more massive than mass B. Mass A is initially moving 12 m/s in the +*x*-direction. Mass B is initially moving 12 m/s in the –*x*-direction. Assuming that the collision is elastic, calculate the final velocity of both masses after the collision. Show that your results are consistent with conservation of momentum and conservation of kinetic energy.

31. Two objects (A and B) of equal mass collide elastically. Mass A is initially moving 5.0 m/s in the +*x*-direction prior to the collision. Mass B is initially moving 3.0 m/s in the –*x*-direction prior to the collision. After the collision, mass A will be moving with a velocity of 3.0 m/s in the –*x*-direction. What will be the velocity of mass B after the collision?

a. 3.0 m/s in the +*x*-direction
b. 5.0 m/s in the +*x*-direction
c. 3.0 m/s in the –*x*-direction
d. 5.0 m/s in the –*x*-direction

32. Two objects (A and B) of equal mass collide elastically. Mass A is initially moving 4.0 m/s in the +*x*-direction prior to the collision. Mass B is initially moving 8.0 m/s in the –*x*-direction prior to the collision. After the collision, mass A will be moving with a velocity of 8.0 m/s in the –*x*-direction. (a) Use the principle of conservation of momentum to predict the velocity of mass B after the collision. (b) Use the fact that kinetic energy is conserved in elastic collisions to predict the velocity of mass B after the collision.

33. Two objects of equal mass collide. Object A is initially moving in the +*x*-direction with a speed of 12 m/s, and object B is initially at rest. After the collision, object A is at rest, and object B is moving away with some unknown velocity. There are no external forces acting on the system of two masses. What statement can we make about this collision?

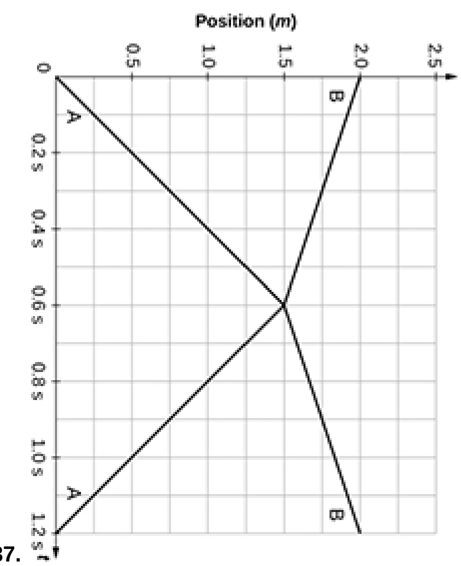
a. Both momentum and kinetic energy are conserved.
b. Momentum is conserved, but kinetic energy is not conserved.
c. Neither momentum nor kinetic energy is conserved.
d. More information is needed in order to determine which is conserved.

34. Two objects of equal mass collide. Object A is initially moving with a velocity of 15 m/s in the +*x*-direction, and object B is initially at rest. After the collision, object A is at rest. There are no external forces acting on the system of two masses. (a) Use momentum conservation to deduce the velocity of object B after the collision. (b) Is this collision elastic? Justify your answer.

35. Which of the following statements is true about an inelastic collision?

a. Momentum is conserved, and kinetic energy is conserved.
b. Momentum is conserved, and kinetic energy is not conserved.
c. Momentum is not conserved, and kinetic energy is conserved.
d. Momentum is not conserved, and kinetic energy is not conserved.

36. Explain how the momentum and kinetic energy of a system of two colliding objects changes as a result of (a) an elastic collision and (b) an inelastic collision.



37.
This figure shows the positions of two colliding objects measured before, during, and after a collision. Mass A is 1.0 kg. Mass B is 3.0 kg. Which of the following statements is true?

a. This is an elastic collision, with a total momentum of 0 kg • m/s.
b. This is an elastic collision, with a total momentum of 1.67 kg • m/s.
c. This is an inelastic collision, with a total momentum of 0 kg • m/s.
d. This is an inelastic collision, with a total momentum of 1.67 kg • m/s.

38. For the above graph, determine the initial and final momentum for both objects, assuming mass A is 1.0 kg and mass B is 3.0 kg. Also, determine the initial and final kinetic energies for both objects. Based on your results, explain whether momentum is conserved in this collision, and state whether the collision is elastic or inelastic.

39. Mass A (1.0 kg) slides across a frictionless surface with a velocity of 8 m/s in the positive direction. Mass B (3.0 kg) is initially at rest. The two objects collide and stick together. What will be the change in the center-of-mass velocity of the system as a result of the collision?

a. There will be no change in the center-of-mass velocity.
b. The center-of-mass velocity will decrease by 2 m/s.
c. The center-of-mass velocity will decrease by 6 m/s.
d. The center-of-mass velocity will decrease by 8 m/s.

40. Mass A (1.0 kg) slides across a frictionless surface with a velocity of 4 m/s in the positive direction. Mass B (1.0 kg) slides across the same surface in the opposite direction with a velocity of −8 m/s. The two objects collide and stick together after the collision. Predict how the center-of-mass velocity will change as a result of the collision, and explain your prediction. Calculate the center-of-mass velocity of the system both before and after the collision and explain why it remains the same or why it has changed.

8.5 Inelastic Collisions in One Dimension

41. Mass A (2.0 kg) has an initial velocity of 4 m/s in the +*x*-direction. Mass B (2.0 kg) has an initial velocity of 5 m/s in the –*x*-direction. If the two masses have an elastic collision, what will be the final velocities of the masses after the collision?

a. Both will move 0.5 m/s in the –*x*-direction.
b. Mass A will stop; mass B will move 9 m/s in the +*x*-direction.
c. Mass B will stop; mass A will move 9 m/s in the –*x*-direction.
d. Mass A will move 5 m/s in the –*x*-direction; mass B will move 4 m/s in the +*x*-direction.

42. Mass A has an initial velocity of 22 m/s in the +*x*-direction. Mass B is three times more massive than mass A and has an initial velocity of 22 m/s in the –*x*-direction. If the two masses have an elastic collision, what will be the final velocities of the masses after the collision?

43. Mass A (2.0 kg) is moving with an initial velocity of 15 m/s in the +*x*-direction, and it collides with mass B (5.0 kg), initially at rest. After the collision, the two objects stick together and move as one. What is the change in kinetic energy of the system as a result of the collision?

a. no change
b. decrease by 225 J
c. decrease by 161 J
d. decrease by 64 J

44. Mass A (2.0 kg) is moving with an initial velocity of 15 m/s in the +*x*-direction, and it collides with mass B (4.0 kg), initially moving 7.0 m/s in the +*x*-direction. After the collision, the two objects stick together and move as one. What is the change in kinetic energy of the system as a result of the collision?

45. Mass A slides across a rough table with an initial velocity of 12 m/s in the +*x*-direction. By the time mass A collides with mass B (a stationary object with equal mass), mass A has slowed to 10 m/s. After the collision, the two objects stick together and move as one. Immediately after the collision, the velocity of the system is measured to be 5 m/s in the +*x*-direction, and the system eventually slides to a stop.

Which of the following statements is true about this motion?

a. Momentum is conserved during the collision, but it is not conserved during the motion before and after the collision.
b. Momentum is not conserved at any time during this analysis.
c. Momentum is conserved at all times during this analysis.
d. Momentum is not conserved during the collision, but it is conserved during the motion before and after the collision.

46. Mass A is initially moving with a velocity of 12 m/s in the +*x*-direction. Mass B is twice as massive as mass A and is initially at rest. After the two objects collide, the two masses move together as one with a velocity of 4 m/s in the +*x*-direction. Is momentum conserved in this collision?

47. Mass A is initially moving with a velocity of 24 m/s in the +*x*-direction. Mass B is twice as massive as mass A and is initially at rest. The two objects experience a totally inelastic collision. What is the final speed of both objects after the collision?

a. A is not moving; B is moving 24 m/s in the +*x*-direction.
b. Neither A nor B is moving.
c. A is moving 24 m/s in the –*x*-direction. B is not moving.
d. Both A and B are moving together 8 m/s in the +*x*-direction.

48. Mass A is initially moving with some unknown velocity in the +*x*-direction. Mass B is twice as massive as mass A and initially at rest. The two objects collide, and after the collision, they move together with a speed of 6 m/s in the +*x*-direction. (a) Is this collision elastic or inelastic? Explain. (b) Determine the initial velocity of mass A.

49. Mass A is initially moving with a velocity of 2 m/s in the +*x*-direction. Mass B is initially moving with a velocity of 6 m/s in the –*x*-direction. The two objects have equal masses. After they collide, mass A moves with a speed of 4 m/s in the –*x*-direction. What is the final velocity of mass B after the collision?

a. 6 m/s in the +*x*-direction
b. 4 m/s in the +*x*-direction
c. zero
d. 4 m/s in the –*x*-direction

50. Mass A is initially moving with a velocity of 15 m/s in the +*x*-direction. Mass B is twice as massive and is initially moving with a velocity of 10 m/s in the –*x*-direction. The two objects collide, and after the collision, mass A moves with a speed of 15 m/s in the –*x*-direction. (a) What is the final velocity of mass B after the collision? (b) Calculate the change in kinetic energy as a result of the collision, assuming mass A is 5.0 kg.

8.6 Collisions of Point Masses in Two Dimensions

51. Two cars of equal mass approach an intersection. Car A is moving east at a speed of 45 m/s. Car B is moving south at a speed of 35 m/s. They collide inelastically and stick together after the collision, moving as one object. Which of the following statements is true about the center-of-mass velocity of this system?

a. The center-of-mass velocity will decrease after the collision as a result of lost energy (but not drop to zero).
b. The center-of-mass velocity will remain the same after the collision since momentum is conserved.
c. The center-of-mass velocity will drop to zero since the two objects stick together.
d. The magnitude of the center-of-mass velocity will remain the same, but the direction of the velocity will change.

52. Car A has a mass of 2000 kg and approaches an intersection with a velocity of 38 m/s directed to the east. Car B has a mass of 3500 kg and approaches the intersection with a velocity of 53 m/s directed 63° north of east. The two cars collide and stick together after the collision. Will the center-of-mass velocity change as a result of the collision? Explain why or why not. Calculate the center-of-mass velocity before and after the collision.